
Resolving Parameter Dependences for Interval Analysis of Linear Analog Circuits



Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

Firenze, Italy, October, 12th 2006

SMACD '06 – International Workshop on Symbolic Methods and Applications to Circuit Design

Dr. Alexander Dreyer

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Session T1

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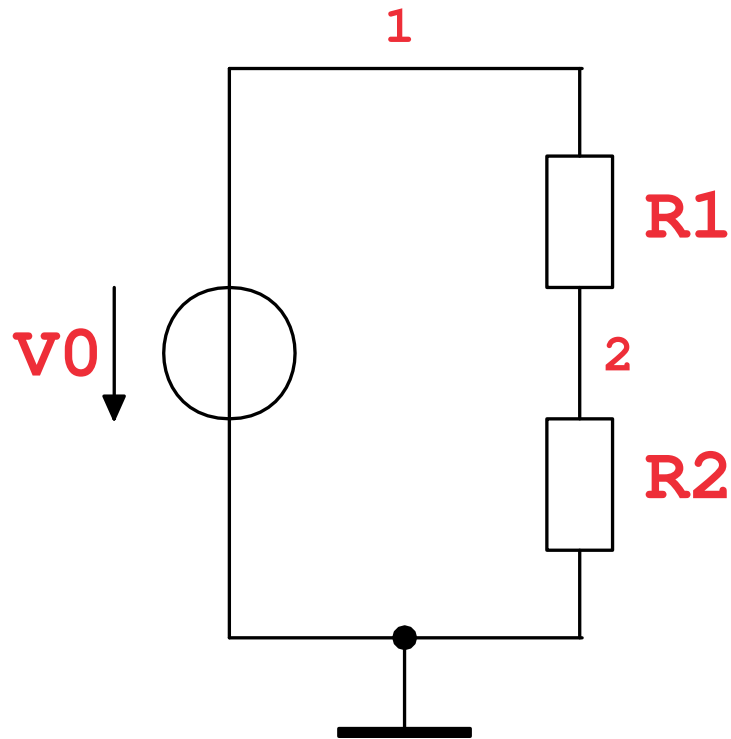
Overview

1. Motivation
2. Fill-in Patterns
3. Resistive Networks
4. RLC Networks
5. Real-world Example
6. Summary



Motivation

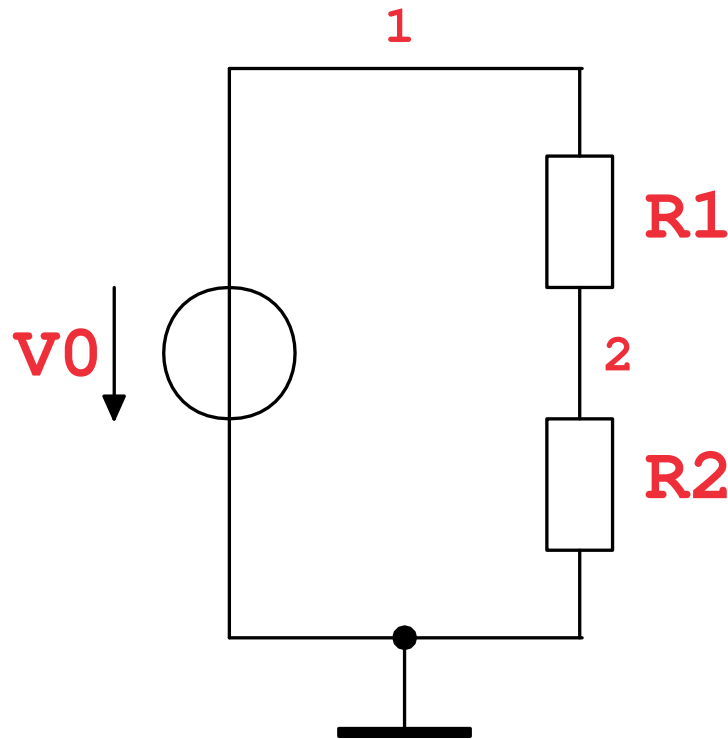
Voltage divider circuit



Motivation

Voltage divider circuit

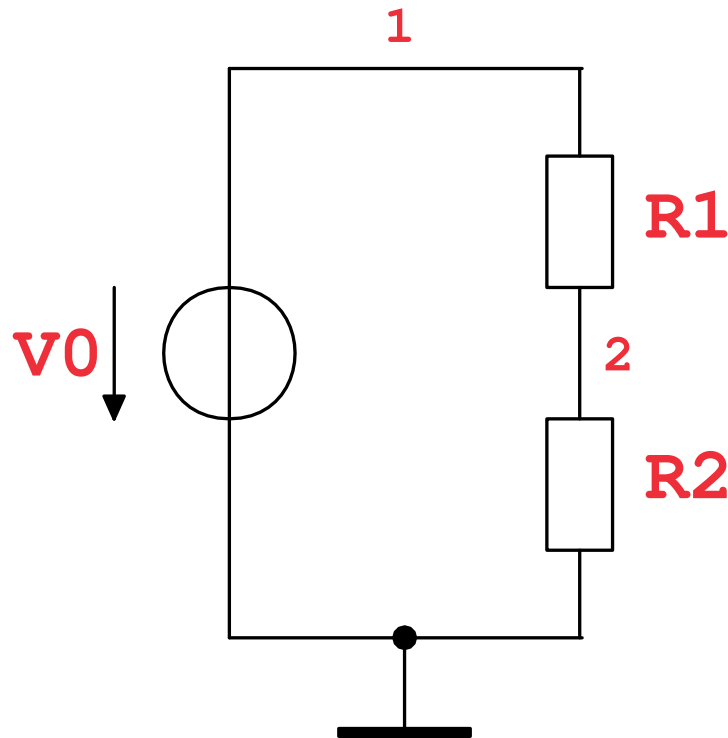
$$\begin{aligned} V_0 &= 1 \text{ V} \\ R_1 &= 10 \Omega \\ R_2 &= 100 \Omega \end{aligned}$$



Motivation

Voltage divider circuit

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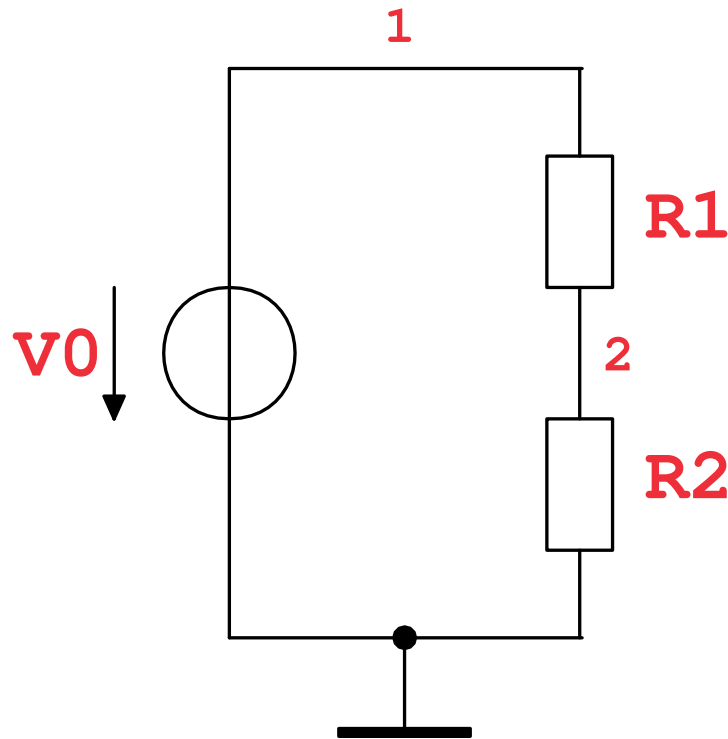
Motivation

Voltage divider circuit

$$\begin{aligned} V_0 &= 1 \text{ V} \\ R_1 &= 10 \Omega \\ R_2 &= 100 \Omega \end{aligned}$$



$$\begin{aligned} V_2 &= \frac{V_0}{R_1/R_2 + 1} \\ &\approx 0.909 \text{ V} \end{aligned}$$



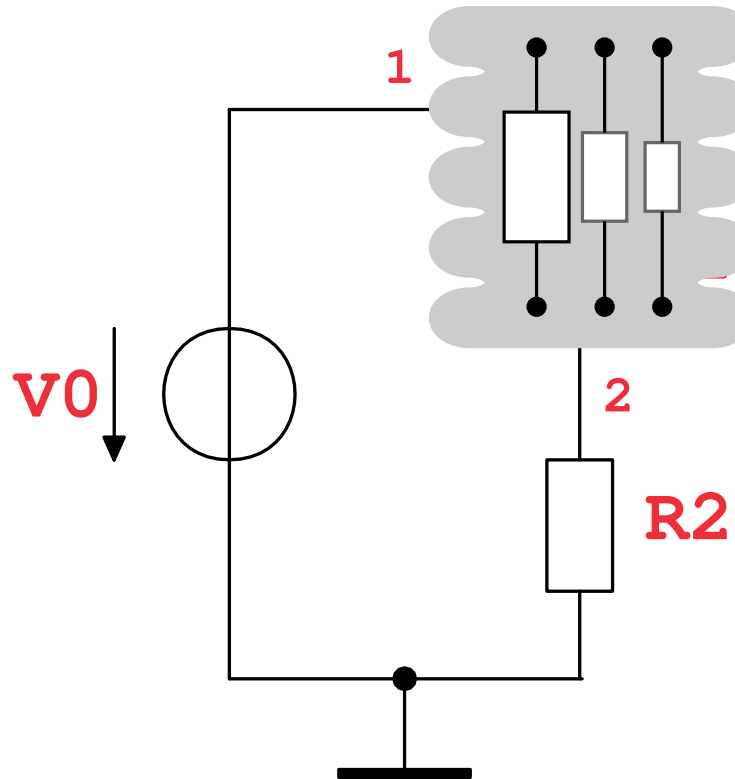
Motivation

Voltage divider circuit
with element tolerances

$$\begin{aligned} V_0 &= 1 \text{ V} \\ R_1 &= 10 \Omega \\ R_2 &= 100 \Omega \end{aligned}$$



$$\begin{aligned} V_2 &= \frac{V_0}{R_1/R_2 + 1} \\ &\approx 0.909 \text{ V} \end{aligned}$$



Motivation

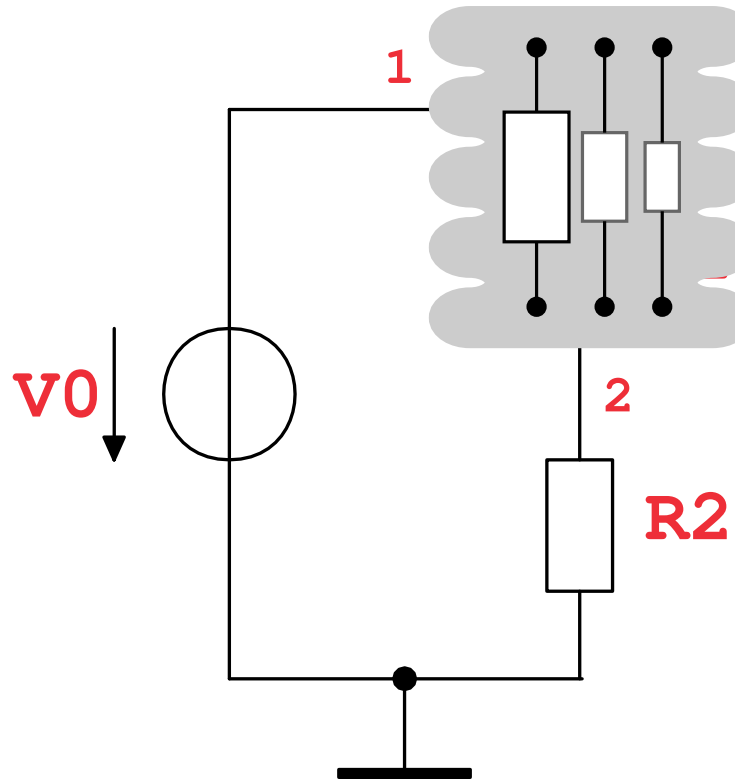
Voltage divider circuit

with element tolerances

$$\begin{aligned} V_0 &= 1\text{ V} \pm 10\% \\ R_1 &= 10\ \Omega \pm 10\% \\ R_2 &= 100\ \Omega \pm 10\% \end{aligned}$$



$$\begin{aligned} V_2 &= \frac{V_0}{R_1/R_2 + 1} \\ &\approx 0.909\text{ V} \pm ? \end{aligned}$$



Motivation

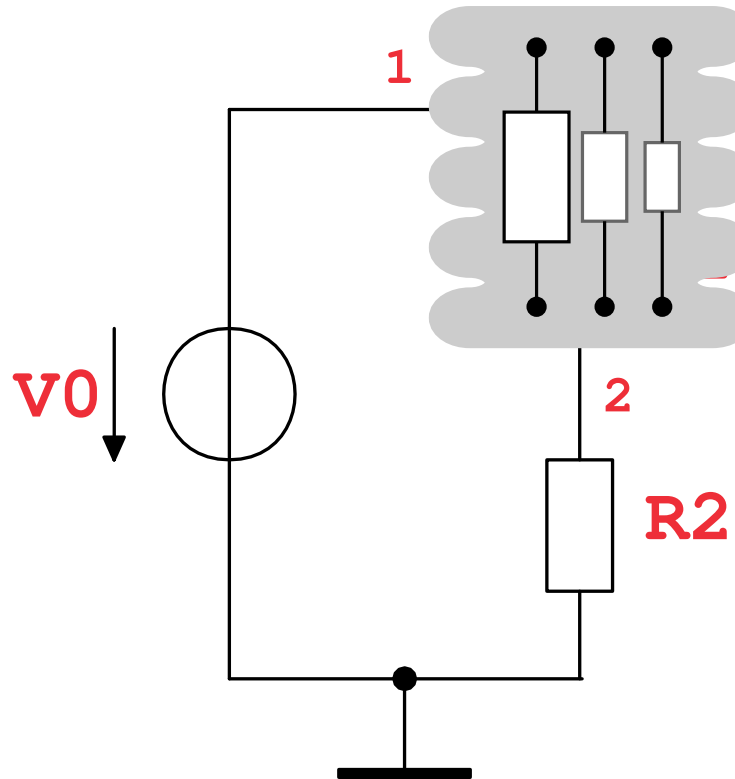
Voltage divider circuit

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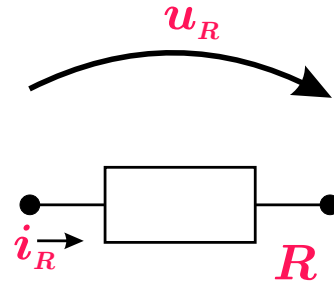
$$\begin{aligned} V_2 &= \frac{[0.9, 1.1]}{[9, 11]/[90, 110] + 1} \\ &\in [0.80, 1.02] \end{aligned}$$



worst case analysis
using interval-arithmetic

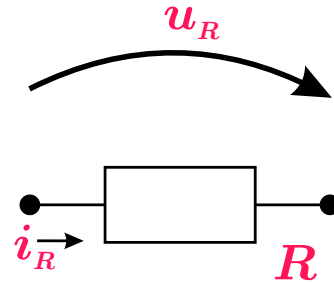
Fill-in Patterns

Resistor



Fill-in Patterns

Resistor

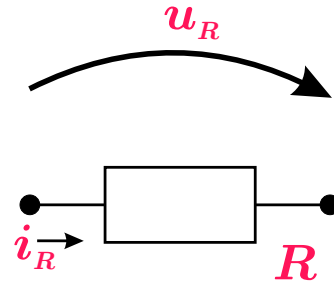


Sparse tableau analysis

$$\begin{pmatrix} \dots & \vdots & \dots & \vdots & \dots \\ \dots & -1 & \dots & \frac{1}{R} & \dots \\ \dots & \vdots & \dots & \vdots & \dots \end{pmatrix} \cdot \begin{pmatrix} \vdots \\ i_R \\ \vdots \\ u_R \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \end{pmatrix}$$

Fill-in Patterns

Resistor



Sparse tableau analysis

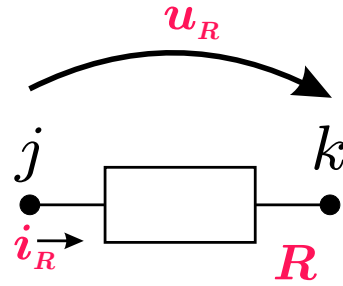
$$\begin{pmatrix} \vdots & & \vdots & & \vdots & & \vdots \\ \dots & -1 & \dots & \frac{1}{R} & \dots & & \dots \\ \vdots & & \vdots & & \vdots & & \vdots \end{pmatrix} \cdot \begin{pmatrix} \vdots \\ i_R \\ \vdots \\ u_R \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \end{pmatrix}$$

Superposition of matrices

$$p_i \cdot \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \cdot (0 \quad \dots \quad 0 \quad 1 \quad 0 \quad \dots \quad 0)$$

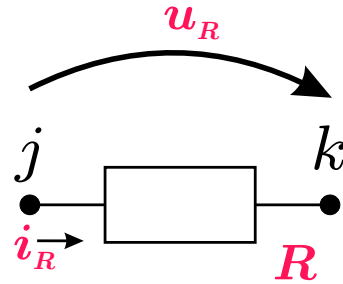
Fill-in Patterns

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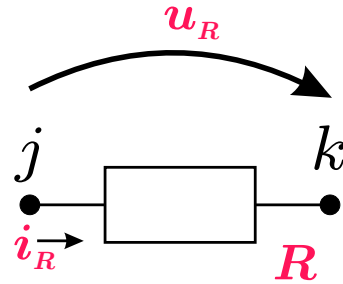


Modified nodal analysis

$$\begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & -1 & \dots & 0 & \dots & \frac{1}{R} & \dots & -\frac{1}{R} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & 0 & \dots & -1 & \dots & -\frac{1}{R} & \dots & \frac{1}{R} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \cdot \begin{pmatrix} \vdots \\ i_j \\ \vdots \\ i_k \\ \vdots \\ u_j \\ \vdots \\ u_k \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \end{pmatrix}$$

Fill-in Patterns

Resistor



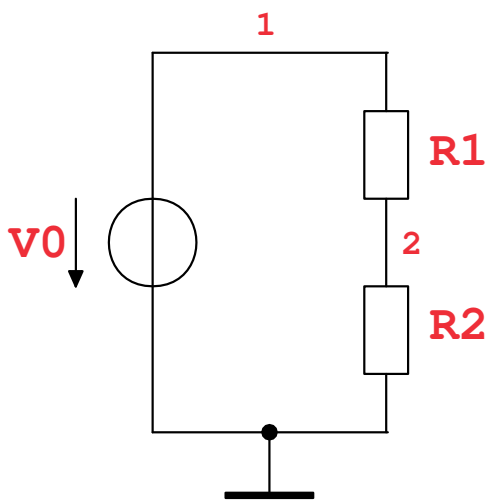
Modified nodal analysis

$$\begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & -1 & \dots & 0 & \dots & \frac{1}{R} & \dots & -\frac{1}{R} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & 0 & \dots & -1 & \dots & -\frac{1}{R} & \dots & \frac{1}{R} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \cdot \begin{pmatrix} \vdots \\ i_j \\ \vdots \\ i_k \\ \vdots \\ u_j \\ \vdots \\ u_k \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \end{pmatrix}$$

General case

$$\left(\mathbf{A}_0 + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) \right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}_0 + \sum_j s_j \cdot \mathbf{r}_j$$

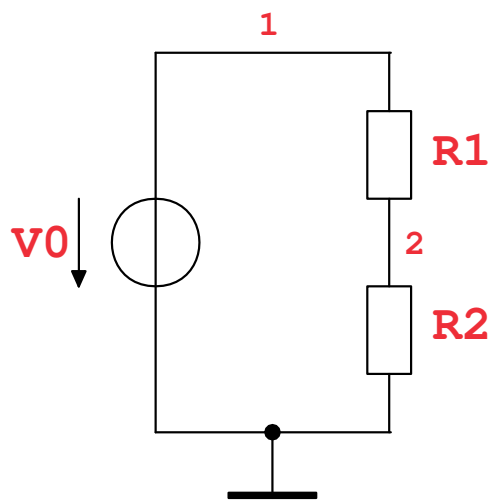
Resistive Networks



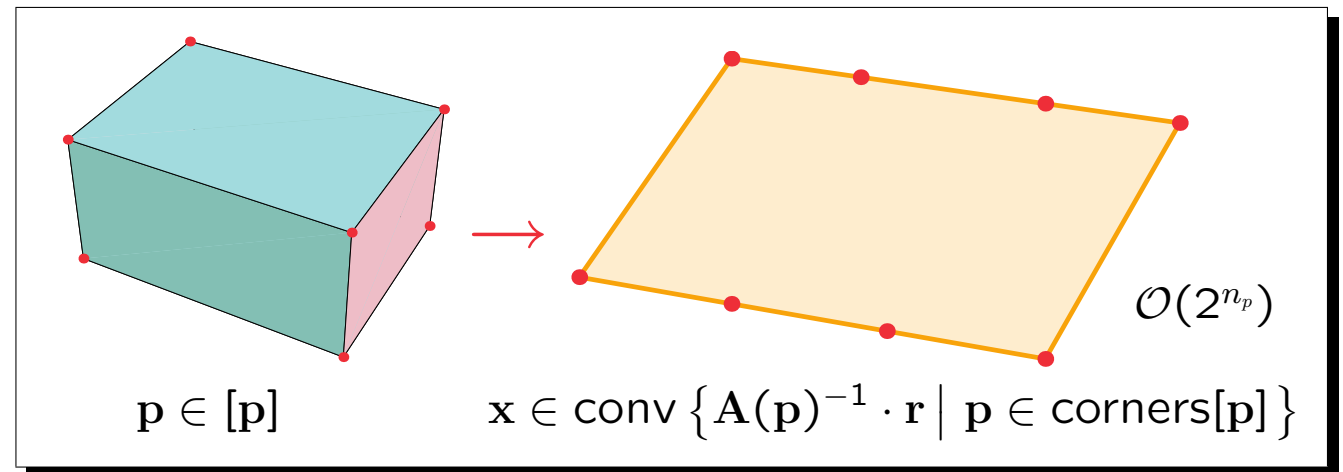
$$\left(A_0 + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) \right) \cdot \mathbf{x} = \mathbf{r}$$

$$p_i \in [-\Delta p_i, \Delta p_i]$$

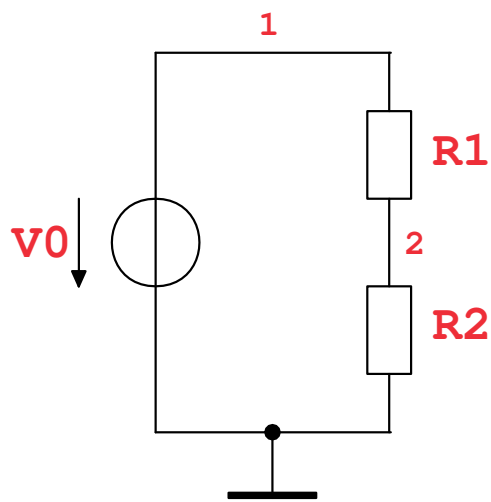
Resistive Networks



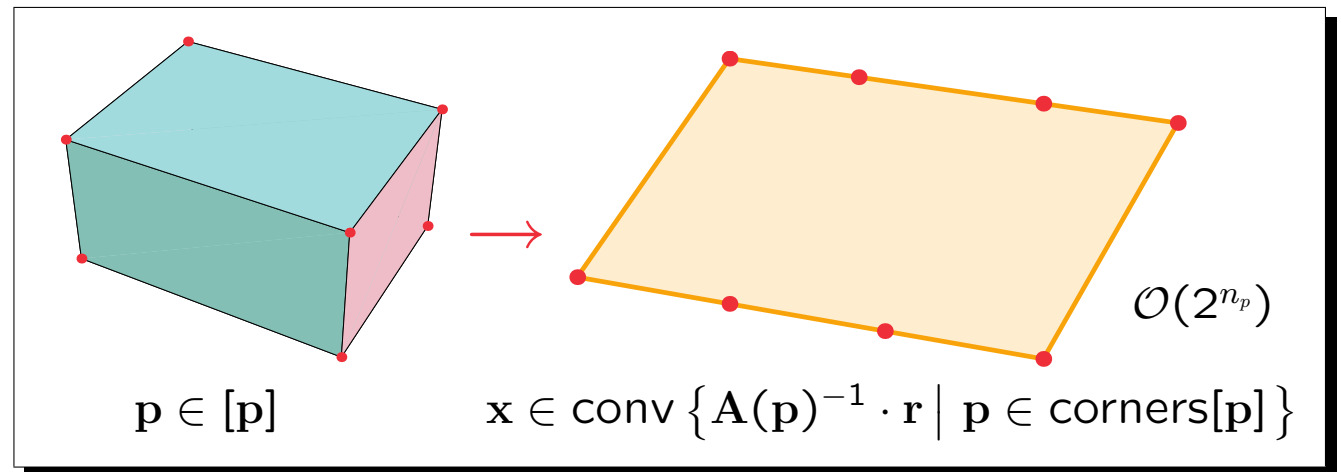
$$\left(\mathbf{A}_0 + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) \right) \cdot \mathbf{x} = \mathbf{r} \quad p_i \in [-\Delta p_i, \Delta p_i]$$



Resistive Networks



$$\left(\mathbf{A}_0 + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) \right) \cdot \mathbf{x} = \mathbf{r} \quad p_i \in [-\Delta p_i, \Delta p_i]$$



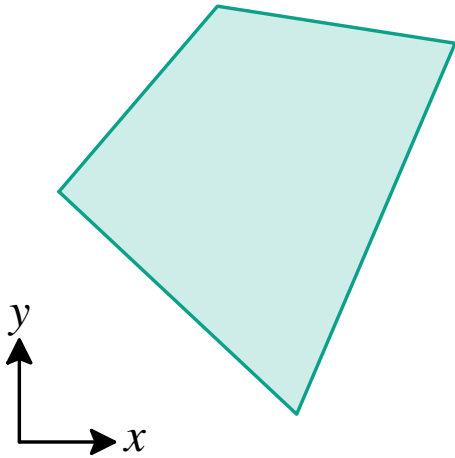
Regularity condition

$$0 \neq \text{sign det } \mathbf{A}(\mathbf{p}) = \text{const}$$

for $\mathbf{p} \in \text{corners}[p]$

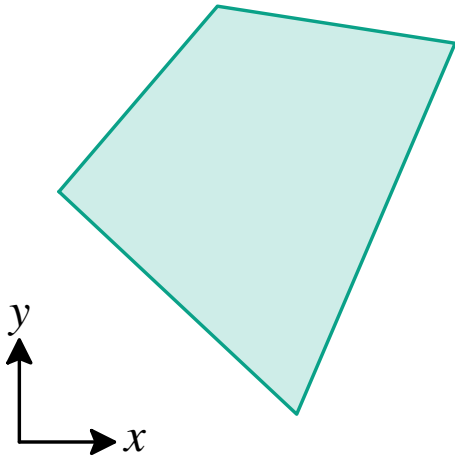


Resistive Networks: Quick variant



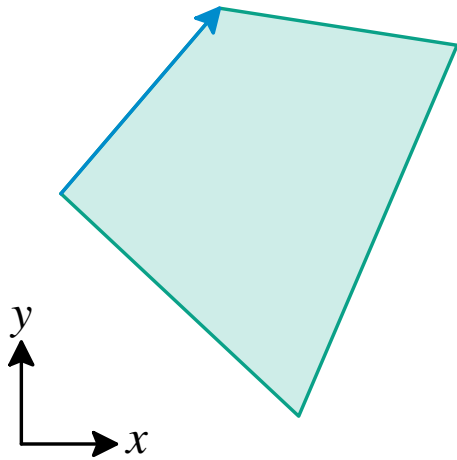
$$\left(\mathbf{A}_0 + \sum_{i \neq j} p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\top) + p_j \cdot (\mathbf{u}_j \cdot \mathbf{v}_j^\top) \right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$
$$p_i \in [-\Delta p_i, \Delta p_i]$$

Resistive Networks: Quick variant



$$\underbrace{\left(\mathbf{A}_0 + \sum_{i \neq j} p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\top) + p_j \cdot (\mathbf{u}_j \cdot \mathbf{v}_j^\top) \right)}_{\mathbf{B}} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r} \quad p_i \in [-\Delta p_i, \Delta p_i]$$

Resistive Networks: Quick variant

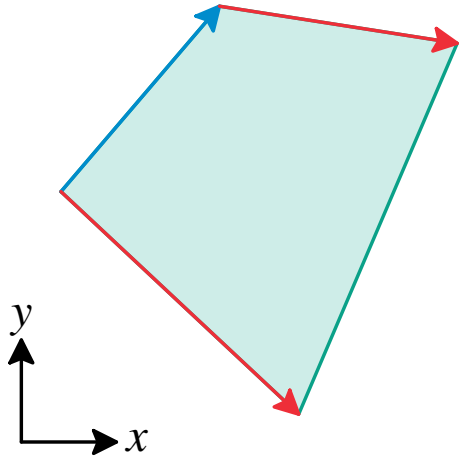


Sherman-Morrison formula

$$\underbrace{\left(\mathbf{A}_0 + \sum_{i \neq j} p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\top) + p_j \cdot (\mathbf{u}_j \cdot \mathbf{v}_j^\top) \right)}_{\mathbf{B}} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r} \quad p_i \in [-\Delta p_i, \Delta p_i]$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^\top \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^\top \mathbf{B}^{-1} \right) \mathbf{r}$$

Resistive Networks: Quick variant

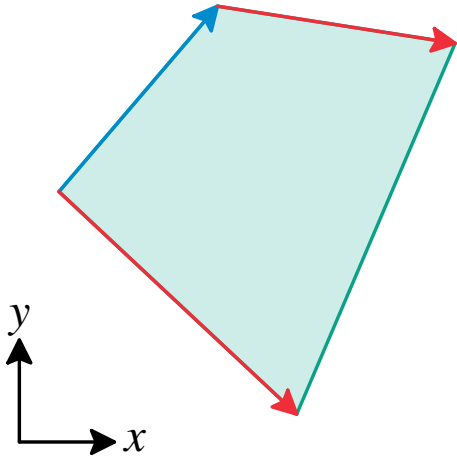


Sherman-Morrison formula

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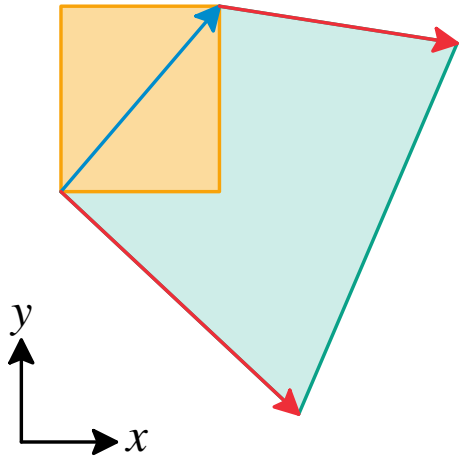
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Resistive Networks: Quick variant



$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^T \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^T \mathbf{B}^{-1} \right) \mathbf{r}$$

Resistive Networks: Quick variant



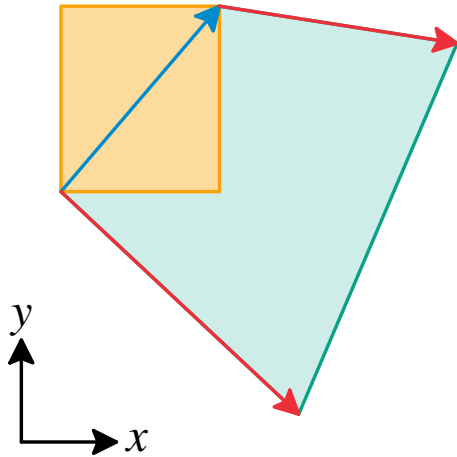
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$$[\tilde{\mathbf{r}}] := \mathbf{A}_0^{-1} \cdot \mathbf{r}$$

$$[\tilde{\mathbf{u}}_j] := \mathbf{A}_0^{-1} \cdot \mathbf{u}_j$$

$$[y] = 1/[p_j] + \mathbf{v}_j^T \cdot [\tilde{\mathbf{u}}_j]$$

Resistive Networks: Quick variant



$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^T \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^T \mathbf{B}^{-1} \right) \mathbf{r}$$

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$$[y] = 1/[p_j] + \mathbf{v}_j^T \cdot [\tilde{\mathbf{u}}_j]$$

If $[y] \not\equiv 0$

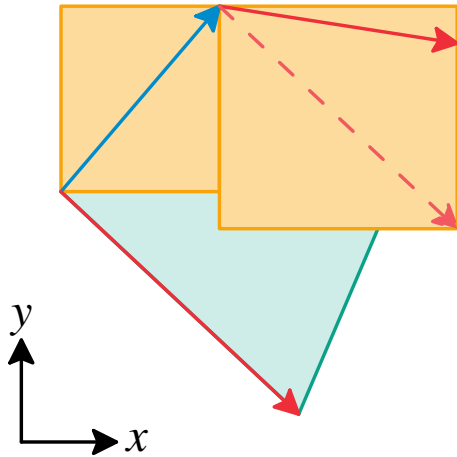
$$[\tilde{\mathbf{r}}] \leftarrow [\tilde{\mathbf{r}}] - \frac{1}{[y]} \cdot ([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^T \cdot [\tilde{\mathbf{r}}]))$$

Regularity test

Interval-valued

Sherman-Morrison

Resistive Networks: Quick variant



$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^T \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^T \mathbf{B}^{-1} \right) \mathbf{r}$$

Initializing

$$[\tilde{\mathbf{r}}] := \mathbf{A}_0^{-1} \cdot \mathbf{r}$$

$$[\tilde{\mathbf{u}}_j] := \mathbf{A}_0^{-1} \cdot \mathbf{u}_j, \quad j = 1, \dots, n_p$$

For $j = 1, \dots, n_p$

$$[y] = 1/[p_j] + \mathbf{v}_j^T \cdot [\tilde{\mathbf{u}}_j]$$

If $[y] \neq 0$

$$[\tilde{\mathbf{r}}] \leftarrow [\tilde{\mathbf{r}}] - \frac{1}{[y]} \cdot ([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^T \cdot [\tilde{\mathbf{r}}]))$$

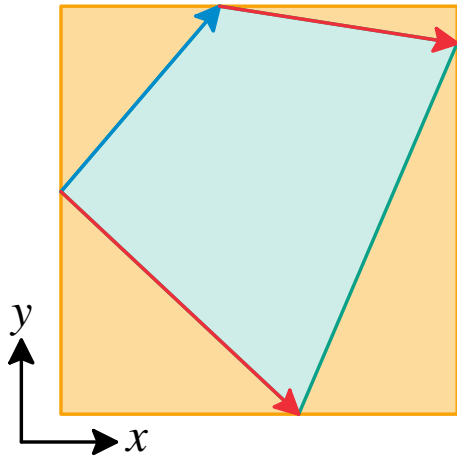
$$[\tilde{\mathbf{u}}_i] \leftarrow [\tilde{\mathbf{u}}_i] - \frac{1}{[y]} \cdot ([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^T \cdot [\tilde{\mathbf{u}}_i])), \quad i > j$$

Regularity test

Interval-valued

Sherman-Morrison

Resistive Networks: Quick variant



$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^T \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^T \mathbf{B}^{-1} \right) \mathbf{r}$$

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$$[\tilde{\mathbf{r}}] := \mathbf{A}_0^{-1} \cdot \mathbf{r}$$

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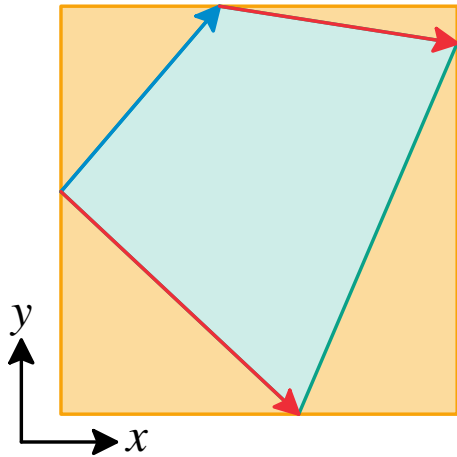
$$[\tilde{\mathbf{u}}_i] \leftarrow [\tilde{\mathbf{u}}_i] - \frac{1}{[y]} \cdot ([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^T \cdot [\tilde{\mathbf{u}}_i])), \quad i > j$$

Regularity test

Interval-valued

Sherman-Morrison

Resistive Networks: Quick variant



$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^T \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^T \mathbf{B}^{-1} \right) \mathbf{r}$$

Initializing

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$$[\tilde{\mathbf{u}}_i] \leftarrow [\tilde{\mathbf{u}}_i] - \frac{1}{[y]} \cdot ([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^T \cdot [\tilde{\mathbf{u}}_i])), \quad i > j$$

$\mathcal{O}(n_p^2)$ "payed" by
loss of accuracy

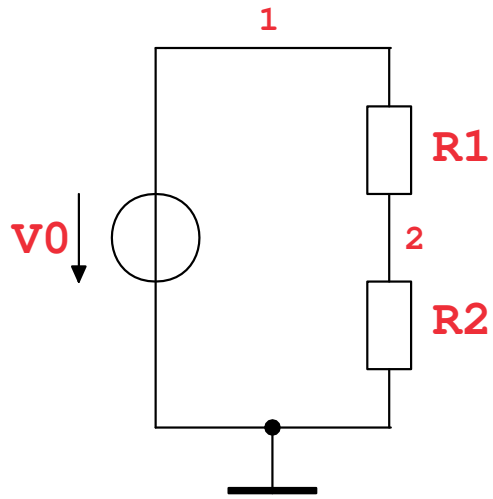
- **Wrapping effect**
 $[\mathbf{x}] \supseteq \text{conv}\{\mathbf{x}_\nu\}_\nu$
- **Dependencies**
 $[\tilde{\mathbf{u}}_\nu] \leftrightarrow [\tilde{\mathbf{u}}_\mu]$

Regularity test

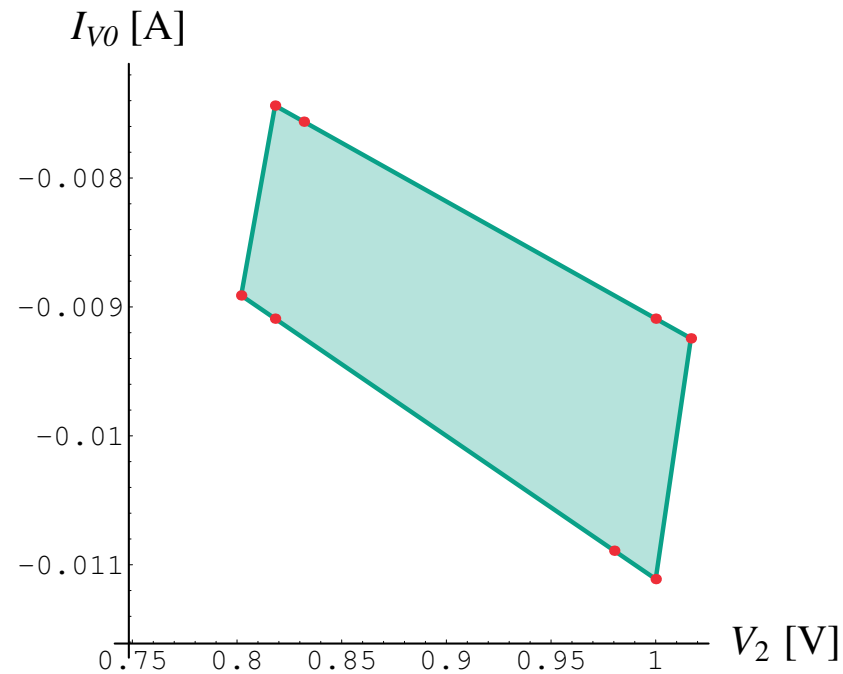
Interval-valued

Sherman-Morrison

Resistive Networks

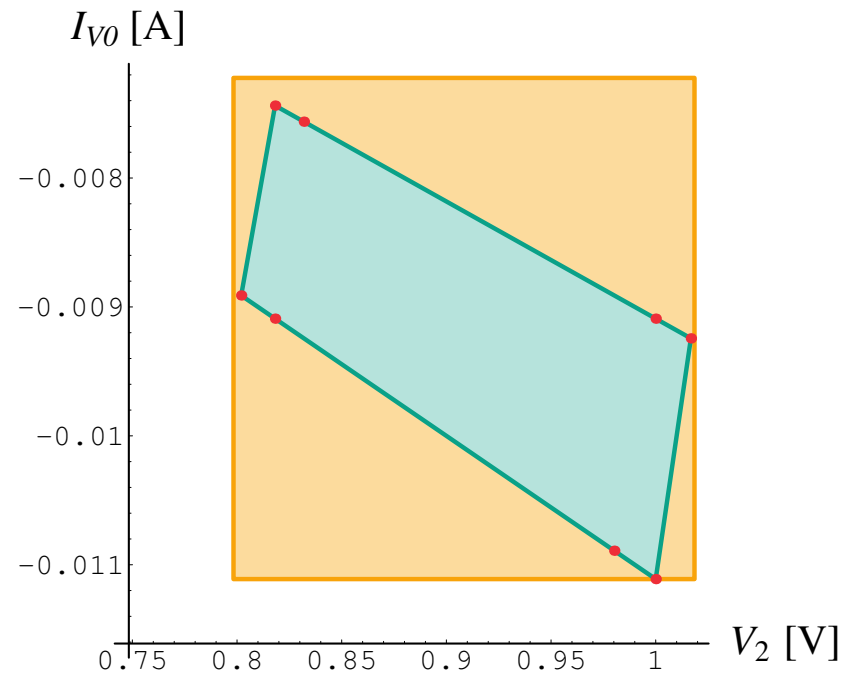
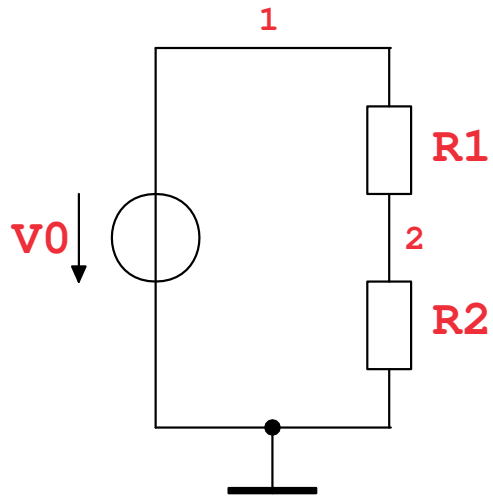


```
conv(...)
```



$V_0 = 1\text{ V}$
 $R_1 = 10\ \Omega$
 $R_2 = 100\ \Omega$
 Tolerance $\pm 10\%$

Resistive Networks

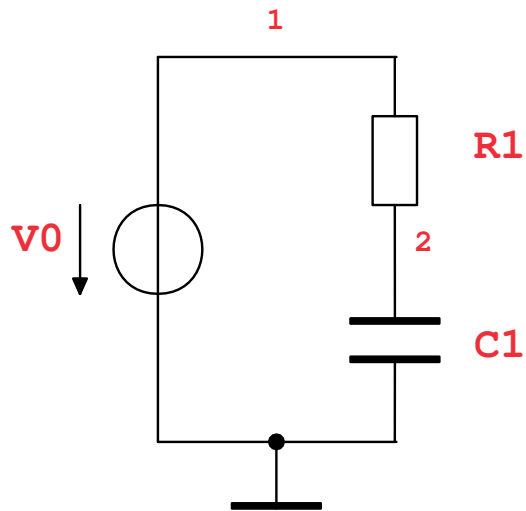


$V_0 = 1 \text{ V}$
 $R_1 = 10 \Omega$
 $R_2 = 100 \Omega$
 Tolerance $\pm 10\%$

— conv(.....)
— Quick variant

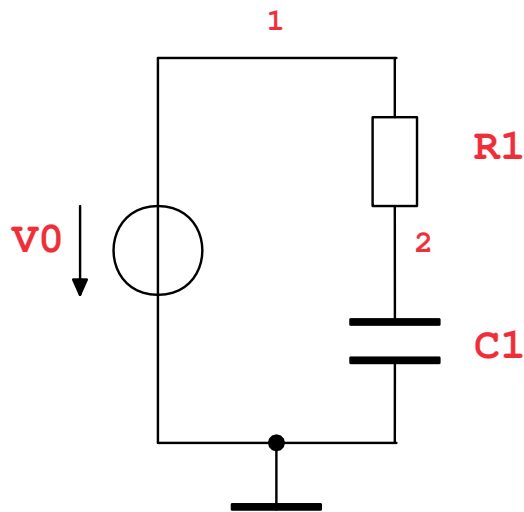


RLC Networks



$$\begin{pmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \frac{1}{R_1} + C_1 s & 0 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ I_{V_0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_0 \end{pmatrix} \quad s = 2\pi i f \in \mathbb{C}$$

RLC Networks



$$\begin{pmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \frac{1}{R_1} + C_1 s & 0 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ I_{V_0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_0 \end{pmatrix} \quad s = 2\pi i f \in \mathbb{C}$$



real representation

$$\begin{pmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} & 0 & 0 & -C_1 2\pi f & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ 0 & C_1 2\pi f & 0 & -\frac{1}{R_1} & \frac{1}{R_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \operatorname{Re} V_1 \\ \operatorname{Re} V_2 \\ \operatorname{Re} I_{V_0} \\ \operatorname{Im} V_1 \\ \operatorname{Im} V_2 \\ \operatorname{Im} I_{V_0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

RLC Networks

Fill-in patterns

$$\left(\mathbf{A} + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\top) + \sum_i p_i \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^\top) \right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

Two patterns per parameter



RLC Networks

Fill-in patterns

$$\left(\mathbf{A} + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\top) + \sum_i p_i \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^\top) \right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

Two patterns per parameter

Outer approximation

$$\left(\mathbf{A} + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\top) + \sum_i \check{p}_i \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^\top) \right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

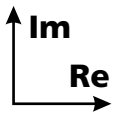
Independent treatment



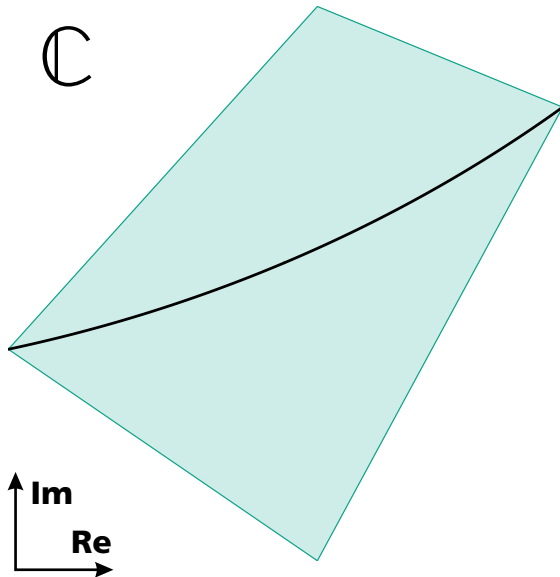
RLC Networks

\mathbb{C}

$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\top) + p \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^\top)) \cdot \mathbf{x} = \mathbf{r}$$



RLC Networks

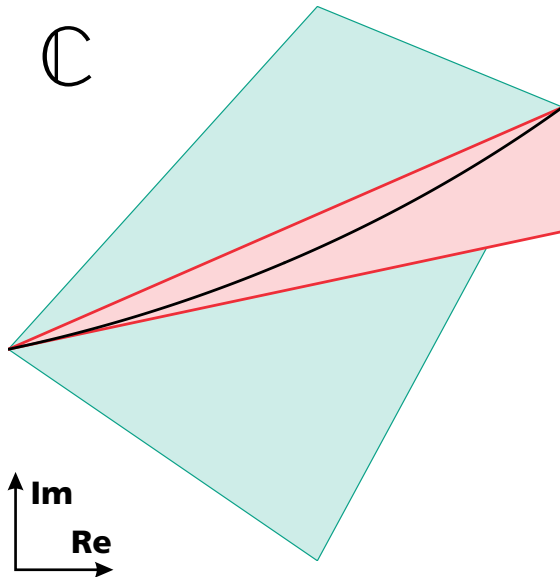
 \mathbb{C}


$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) + q \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^T)) \cdot \mathbf{x} = \mathbf{r}$$

$$\mathbf{A}(p, q) := \mathbf{A}_0 + p \cdot (\mathbf{u} \cdot \mathbf{v}^T) + q \cdot (\check{\mathbf{u}} \cdot \check{\mathbf{v}}^T), \text{ regular } p, q \in [-\Delta p, \Delta p]$$

$$\mathbf{x} \in \text{conv} \left\{ \begin{array}{ccc} & \mathbf{A}(-\Delta p, \Delta p)^{-1} \mathbf{r} & \\ \mathbf{A}(-\Delta p, -\Delta p)^{-1} \mathbf{r} & \text{◇} & \mathbf{A}(\Delta p, \Delta p)^{-1} \mathbf{r} \\ & \mathbf{A}(\Delta p, -\Delta p)^{-1} \mathbf{r} & \end{array} \right\}$$

RLC Networks



Generalized
Sherman-Morrison formula

$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) + q \cdot (\tilde{\mathbf{u}}_i \cdot \tilde{\mathbf{v}}_i^T)) \cdot \mathbf{x} = \mathbf{r}$$

$$\mathbf{A}(p, q) := \mathbf{A}_0 + p \cdot (\mathbf{u} \cdot \mathbf{v}^T) + q \cdot (\tilde{\mathbf{u}} \cdot \tilde{\mathbf{v}}^T), \text{ regular } p, q \in [-\Delta p, \Delta p]$$

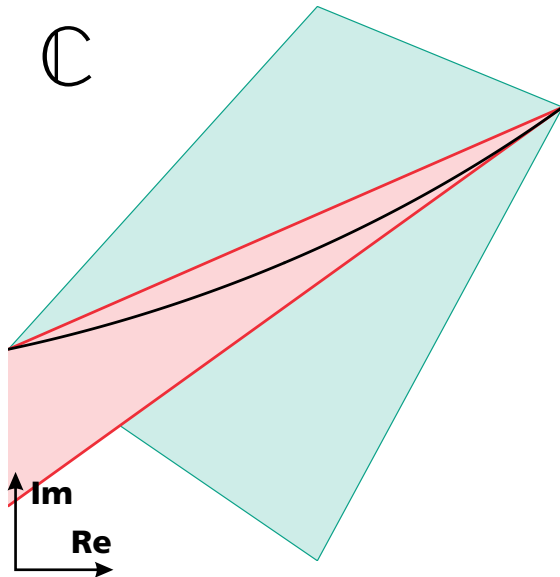
$$\mathbf{x} \in \text{conv} \left\{ \begin{array}{ccc} & \mathbf{A}(-\Delta p, \Delta p)^{-1} \mathbf{r} & \\ \mathbf{A}(-\Delta p, -\Delta p)^{-1} \mathbf{r} & \text{◇} & \mathbf{A}(\Delta p, \Delta p)^{-1} \mathbf{r} \\ & \mathbf{A}(\Delta p, -\Delta p)^{-1} \mathbf{r} & \end{array} \right\}$$

$$\mathbf{x} \in \{ \mathbf{A}(-\Delta p, -\Delta p)^{-1} \mathbf{r} + \lambda \cdot \mathbf{d}_1 + \mu \cdot \mathbf{t}_1 \mid \lambda, \mu \geq 0 \}$$

\mathbf{d}_1 diagonal

\mathbf{t}_1 tangent

RLC Networks



Generalized
Sherman-Morrison formula

$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) + q \cdot (\tilde{\mathbf{u}}_i \cdot \tilde{\mathbf{v}}_i^T)) \cdot \mathbf{x} = \mathbf{r}$$

$$\mathbf{A}(p, q) := \mathbf{A}_0 + p \cdot (\mathbf{u} \cdot \mathbf{v}^T) + q \cdot (\tilde{\mathbf{u}} \cdot \tilde{\mathbf{v}}^T), \text{ regular } p, q \in [-\Delta p, \Delta p]$$

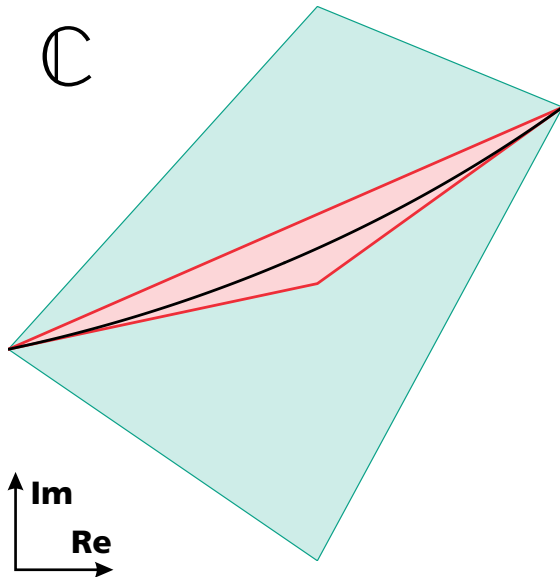
$$\mathbf{x} \in \text{conv} \left\{ \begin{array}{ccc} & \mathbf{A}(-\Delta p, \Delta p)^{-1} \mathbf{r} & \\ \mathbf{A}(-\Delta p, -\Delta p)^{-1} \mathbf{r} & \text{diamond} & \mathbf{A}(\Delta p, \Delta p)^{-1} \mathbf{r} \\ & \mathbf{A}(\Delta p, -\Delta p)^{-1} \mathbf{r} & \end{array} \right\}$$

$$\mathbf{x} \in \{ \mathbf{A}(\Delta p, \Delta p)^{-1} \mathbf{r} + \lambda \cdot \mathbf{d}_2 + \mu \cdot \mathbf{t}_2 \mid \lambda, \mu \geq 0 \}$$

\mathbf{d}_2 diagonal

\mathbf{t}_2 tangent

RLC Networks



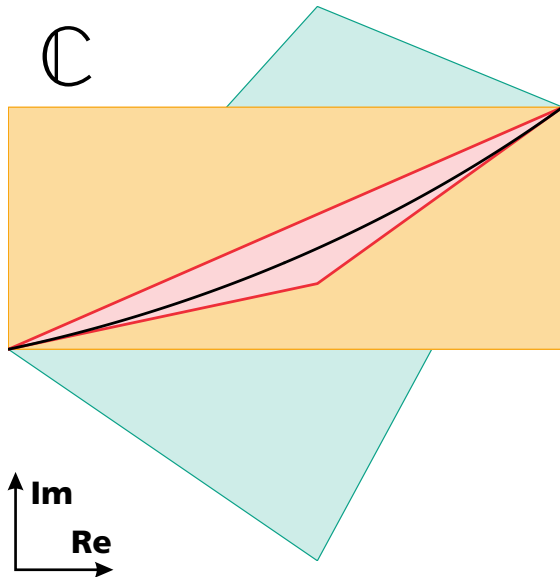
$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) + q \cdot (\tilde{\mathbf{u}}_i \cdot \tilde{\mathbf{v}}_i^T)) \cdot \mathbf{x} = \mathbf{r}$$

$$\mathbf{A}(p, q) := \mathbf{A}_0 + p \cdot (\mathbf{u} \cdot \mathbf{v}^T) + q \cdot (\tilde{\mathbf{u}} \cdot \tilde{\mathbf{v}}^T), \text{ regular } p, q \in [-\Delta p, \Delta p]$$

$$\mathbf{x} \in \text{conv} \left\{ \begin{array}{ccc} & \mathbf{A}(-\Delta p, \Delta p)^{-1} \mathbf{r} & \\ \mathbf{A}(-\Delta p, -\Delta p)^{-1} \mathbf{r} & \text{◇} & \mathbf{A}(\Delta p, \Delta p)^{-1} \mathbf{r} \\ & \mathbf{A}(\Delta p, -\Delta p)^{-1} \mathbf{r} & \end{array} \right\}$$

$$\mathbf{x} \in \text{conv} \left\{ \begin{array}{ccc} & & \mathbf{A}(\Delta p, \Delta p)^{-1} \mathbf{r} \\ \mathbf{A}(-\Delta p, -\Delta p)^{-1} \mathbf{r} & \text{◇} & \\ & \frac{1}{2} (\mathbf{A}(-\Delta p, \Delta p)^{-1} \mathbf{r} + \mathbf{A}(\Delta p, -\Delta p)^{-1} \mathbf{r}) & \end{array} \right\}$$

RLC Networks



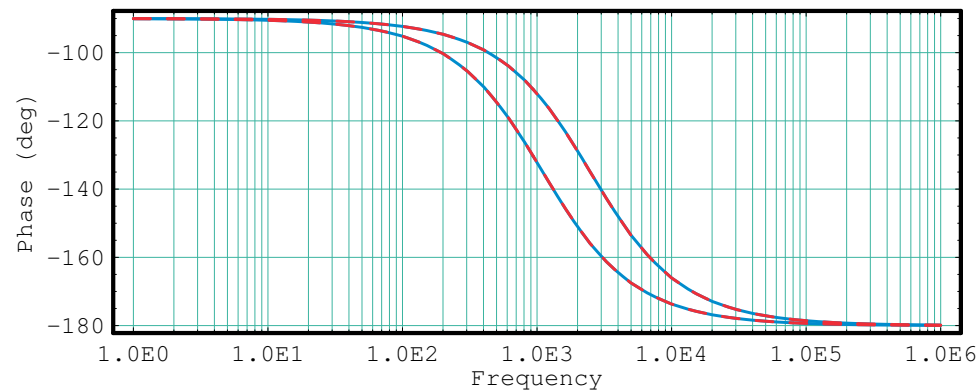
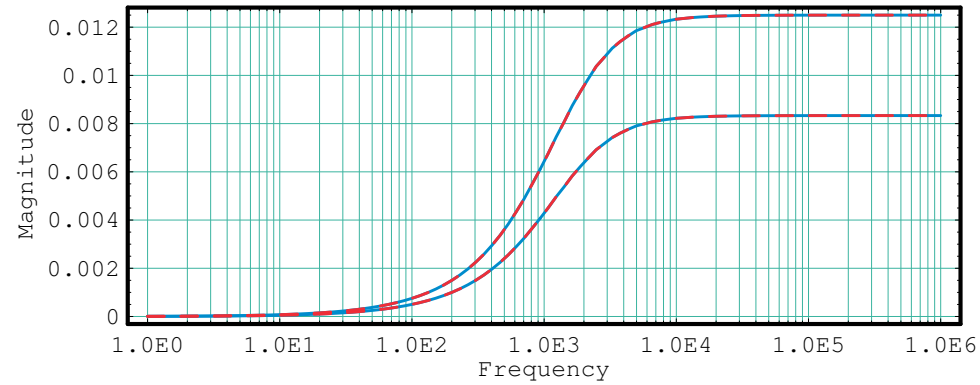
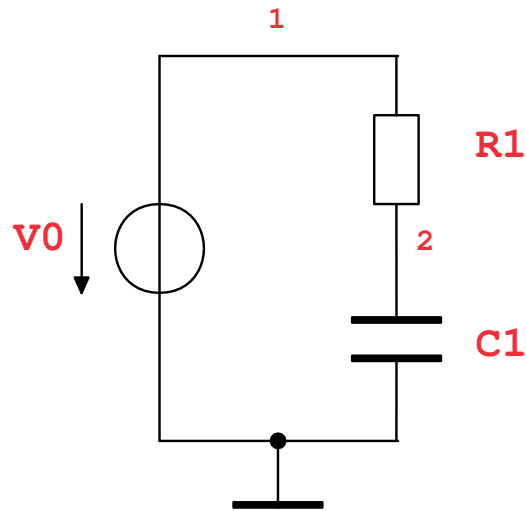
$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^T) + q \cdot (\tilde{\mathbf{u}}_i \cdot \tilde{\mathbf{v}}_i^T)) \cdot \mathbf{x} = \mathbf{r}$$

$$\mathbf{A}(p, q) := \mathbf{A}_0 + p \cdot (\mathbf{u} \cdot \mathbf{v}^T) + q \cdot (\tilde{\mathbf{u}} \cdot \tilde{\mathbf{v}}^T), \text{ regular } p, q \in [-\Delta p, \Delta p]$$

$$\mathbf{x} \in \text{conv} \left\{ \begin{array}{cc} \mathbf{A}(-\Delta p, \Delta p)^{-1} \mathbf{r} & \mathbf{A}(\Delta p, \Delta p)^{-1} \mathbf{r} \\ \mathbf{A}(-\Delta p, -\Delta p)^{-1} \mathbf{r} & \mathbf{A}(\Delta p, -\Delta p)^{-1} \mathbf{r} \end{array} \right\}$$

$$[\mathbf{x}] \supseteq \text{conv} \left\{ \begin{array}{cc} \mathbf{A}(-\Delta p, -\Delta p)^{-1} \mathbf{r} & \mathbf{A}(\Delta p, \Delta p)^{-1} \mathbf{r} \\ \frac{1}{2} (\mathbf{A}(-\Delta p, \Delta p)^{-1} \mathbf{r} + \mathbf{A}(\Delta p, -\Delta p)^{-1} \mathbf{r}) & \end{array} \right\}$$

RLC Networks

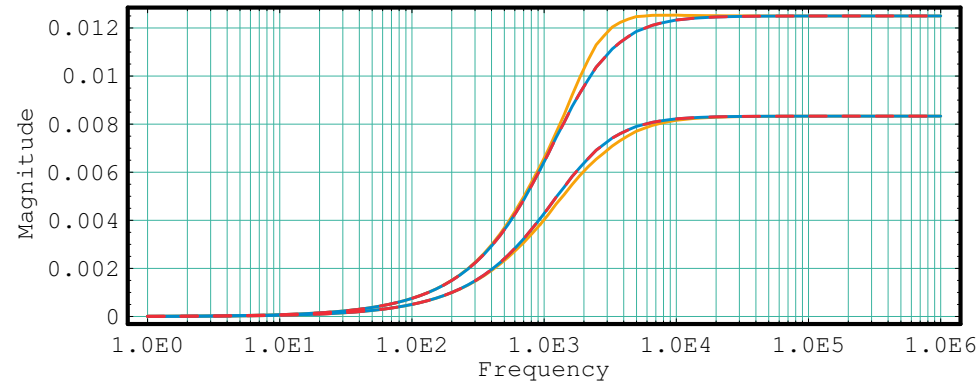
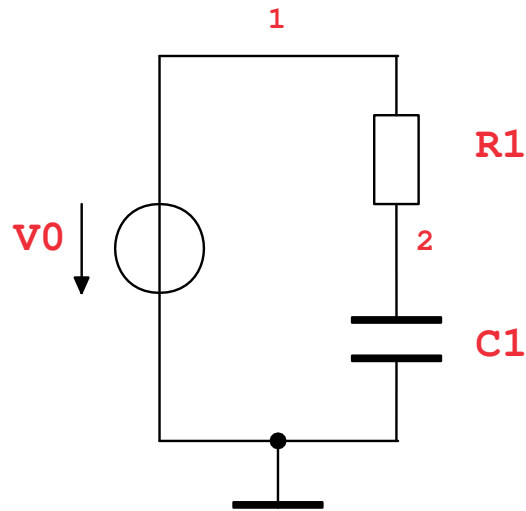


$V1 = 1\text{ V}$
 $R1 = 100\ \Omega$
 $\pm 20\%$
 $C1 = 1\ \mu\text{F}$
 $\pm 20\%$

— Inner approx. for I_{V_0}
— IntervalACAnalysis

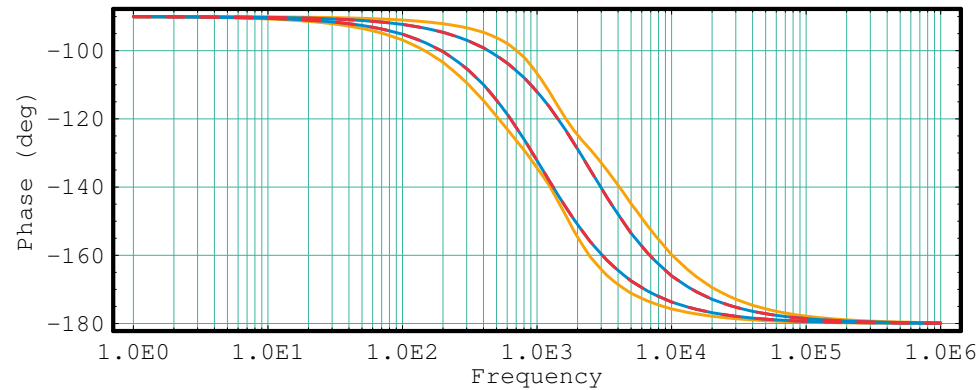


RLC Networks



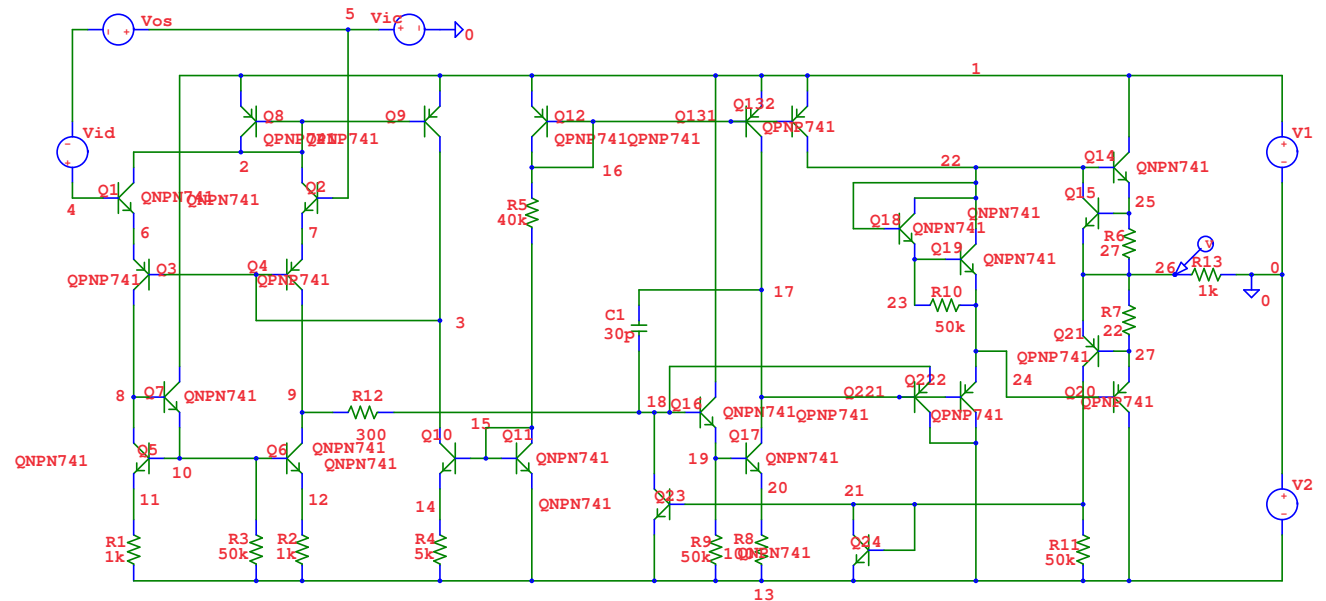
$V_1 = 1 \text{ V}$
 $R_1 = 100 \Omega$
 $\pm 20\%$
 $C_1 = 1 \mu\text{F}$
 $\pm 20\%$

— Inner approx. for I_{V_0}
— IntervalACAnalysis
— QuickACAnalysis



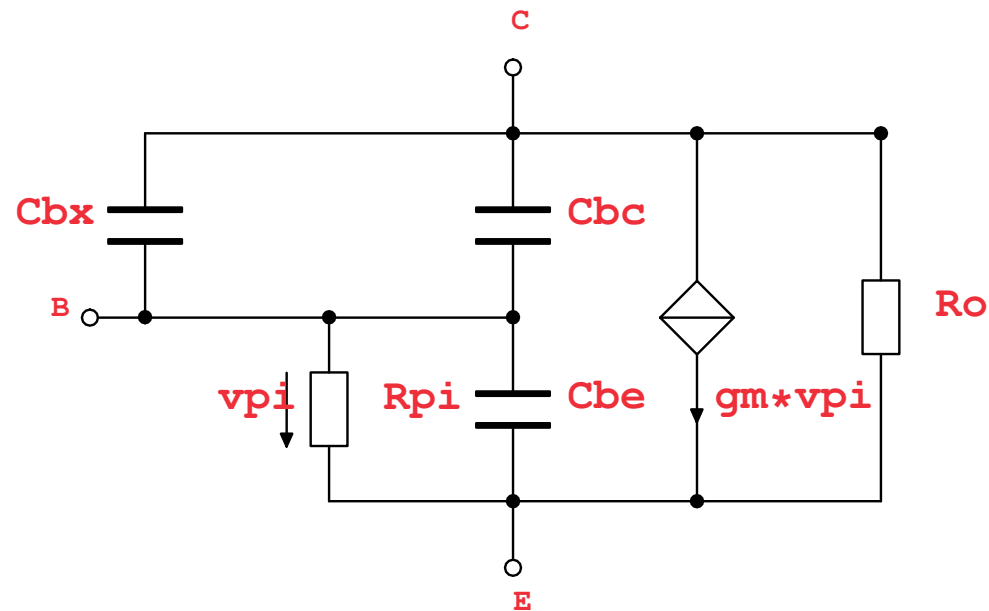
AC-Analysis of an Operational Amplifier

Bound the output voltage for the frequency range from 0.1 Hz to 1 MHz (constant operating point)



AC-Analysis of an Operational Amplifier

Simple linear model
for bipolar transistor



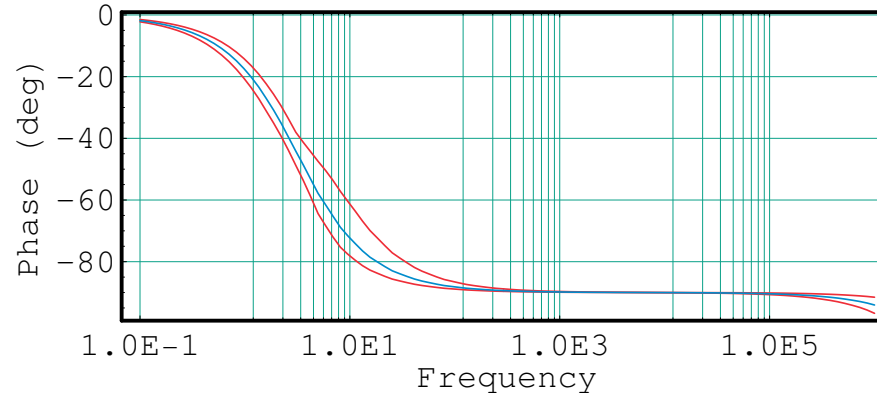
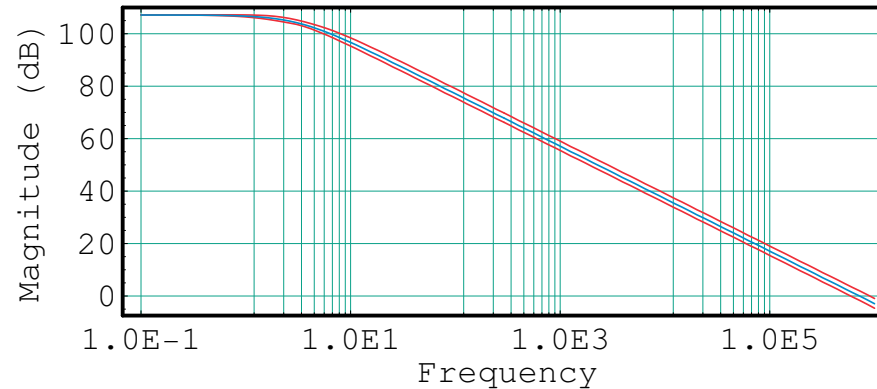
Tolerance specification
(capacitances) $\pm 20\%$

Number of transistors: 26
In sum 79 capacitors (internal and external)

AC-Analysis of an Operational Amplifier

Bound the output voltage for the frequency range from 0.1 Hz to 1 MHz (constant operating point)

Tolerance specification (capacitances) $\pm 20\%$



Summary

Linear circuit analysis

- Method for treatment of large component tolerances and industrial-sized circuits
- Extension to complex-valued systems (frequency domain)
- Support of several circuit formulations (sparse tableau analysis, modified nodal analysis)

Implementation

- Combined symbolic/numerical approach (Mathematica, C++)
- Add-on to ITWM's commercial circuit analysis tool Analog Insydes

