Resolving Parameter Dependences for Interval Analysis of Linear Analog Circuits

Fraunhofer Institut Techno- und Wirtschaftsmathematik

Firenze, Italy, October, 12th 2006

SMACD '06 – International Workshop on Symbolic Methods and Applications to Circuit Design

Dr. Alexander Dreyer

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Session T1

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Fraunhofer-Institut für
Techno- und
Wirtschaftsmathematik

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Overview

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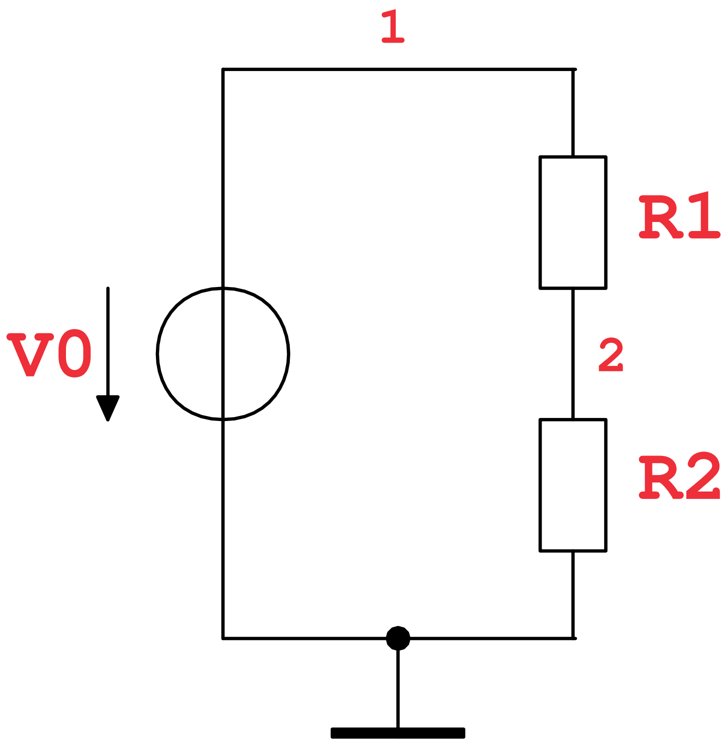
Motivation

Voltage divider circuit



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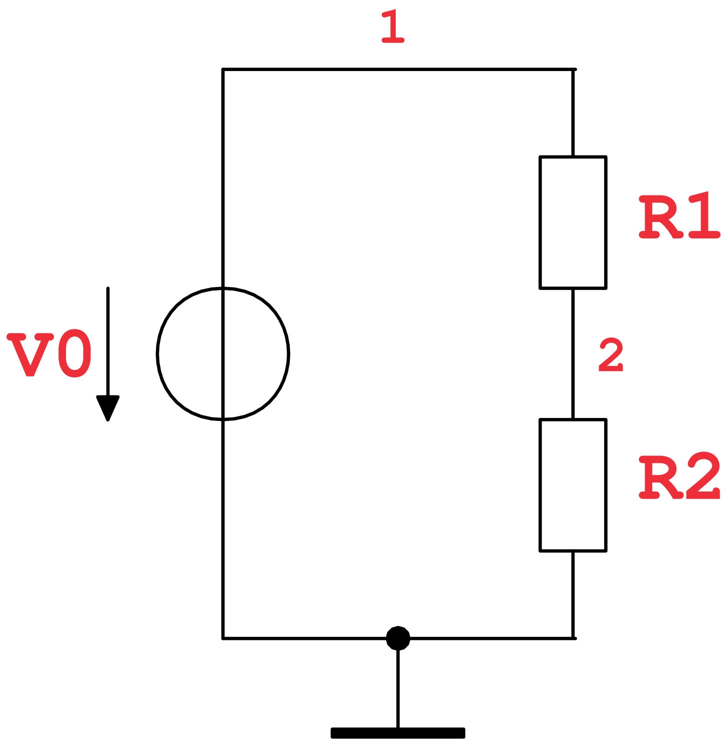
Motivation

Voltage divider circuit

$$\begin{aligned} \mathbf{V0} &= 1\,\mathrm{V} \\ \mathbf{R1} &= 10\,\Omega \\ \mathbf{R2} &= 100\,\Omega \end{aligned}$$



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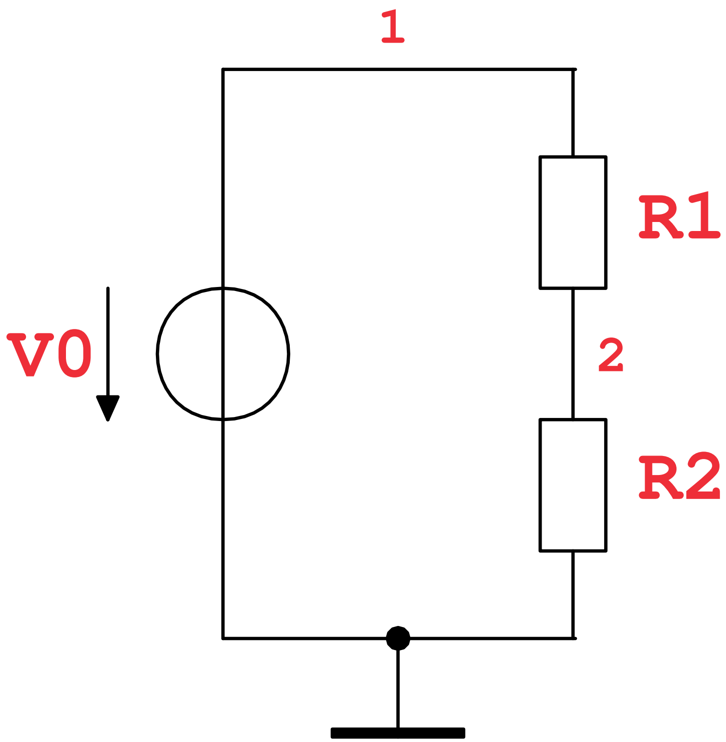
Voltage divider circuit

$$\mathtt{V0} = 1\,\mathrm{V}$$
$$\mathtt{R1} = 10\,\Omega$$
$$\mathtt{R2} = 100\,\Omega$$



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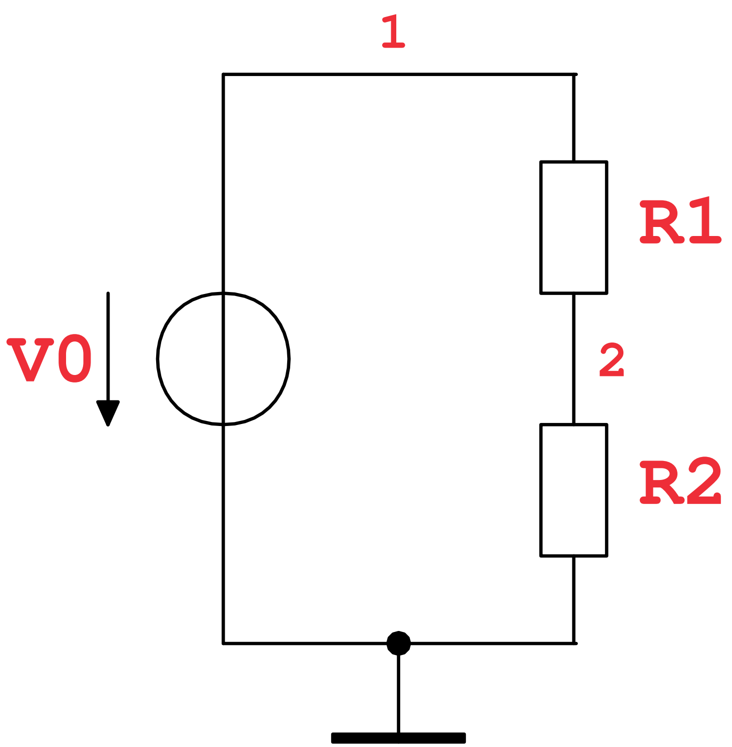
Motivation

Voltage divider circuit

$$V0 = 1\,\mathrm{V}$$
$$R1 = 10\,\Omega$$
$$R2 = 100\,\Omega$$

$$V_2 = \frac{V_0}{R_1/R_2 + 1}$$
$$\approx 0.909\,\mathrm{V}$$



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Motivation

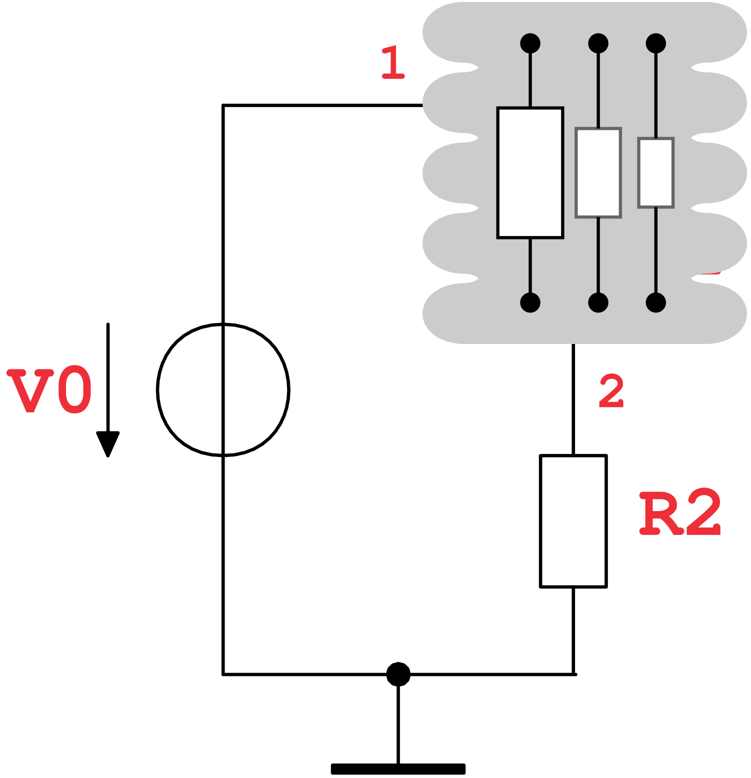
Voltage divider circuit

with element tolerances

$$\begin{aligned} \mathbf{V0} &= 1\,\mathrm{V} \\ \mathbf{R1} &= 10\,\Omega \\ \mathbf{R2} &= 100\Omega \end{aligned}$$

$$V_2 = \frac{V_0}{R_1/R_2 + 1}$$

$$\approx 0.909\,\mathrm{V}$$



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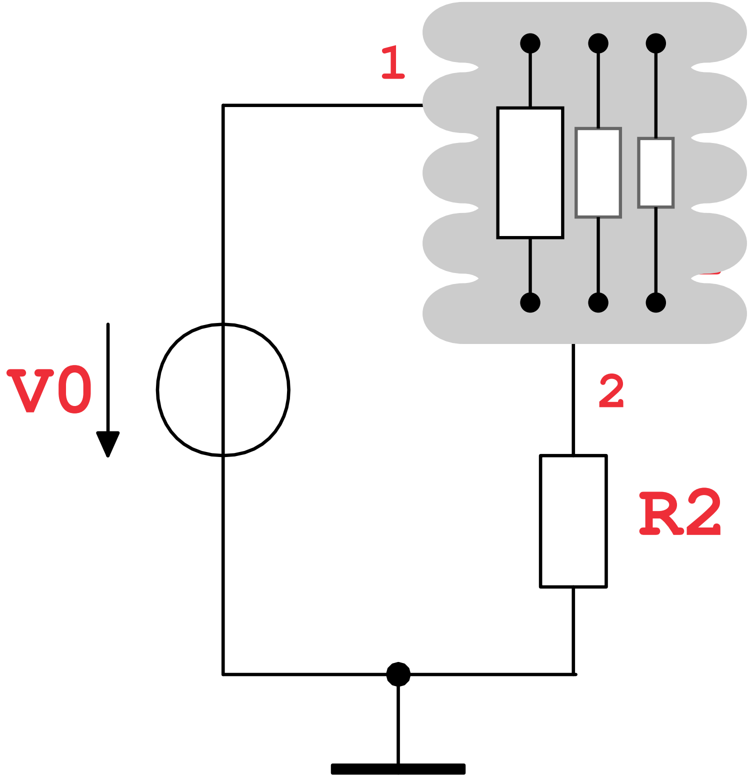
Motivation

Voltage divider circuit

with element tolerances

$$\begin{aligned} \mathbf{V0} &= 1\,\mathrm{V} \pm 10\% \\ \mathbf{R1} &= 10\,\Omega \pm 10\% \\ \mathbf{R2} &= 100\Omega \pm 10\% \end{aligned}$$

$$V_2 = \frac{V_0}{R_1/R_2 + 1} \approx 0.909\,\mathrm{V} \pm ?$$



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Motivation

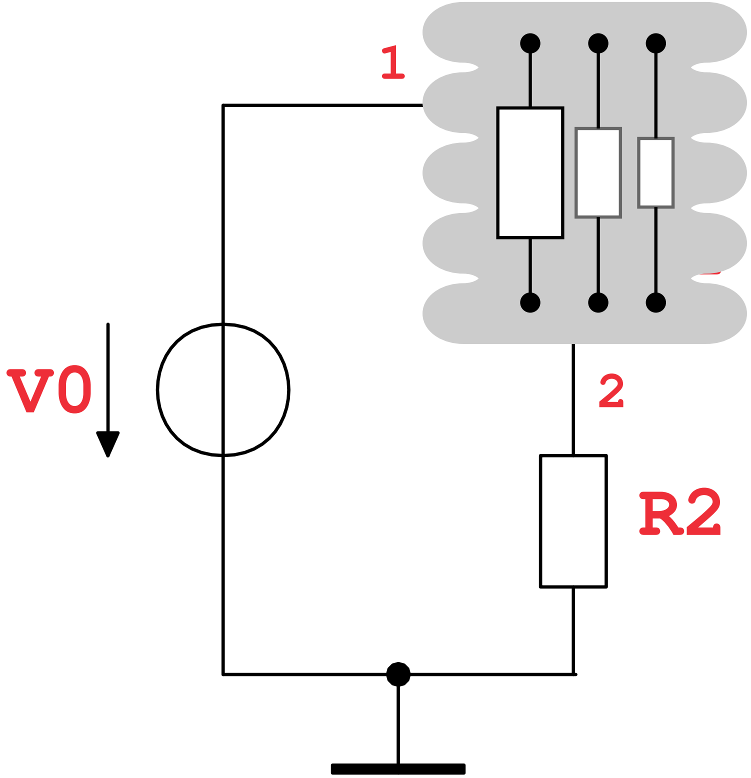
Voltage divider circuit

with element tolerances

$$\begin{aligned} \mathtt{V0} &= 1\,\mathrm{V} \pm 10\% \\ \mathtt{R1} &= 10\,\Omega \pm 10\% \\ \mathtt{R2} &= 100\Omega \pm 10\% \end{aligned}$$

$$V_2 = \frac{[0.9,1.1]}{[9,11]/[90,110]+1}$$

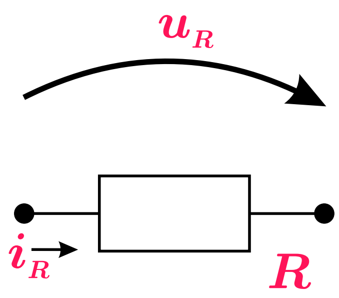
$$\in \; [0.80, 1.02]$$



worst case analysis
using interval-arithmetic

Fill-in Patterns

Resistor



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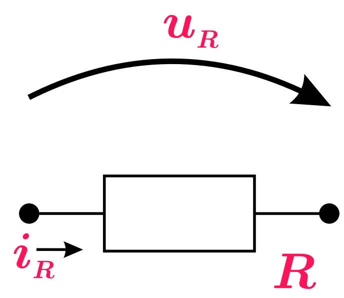
Fill-in Patterns

Resistor



Sparse tableau analysis

$$\begin{pmatrix} & \vdots & & \vdots & \\ \dots & -1 & \dots & \frac{1}{R} & \dots \\ & \vdots & & \vdots & \end{pmatrix} \cdot \begin{pmatrix} \vdots \\ i_R \\ \vdots \\ u_R \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \end{pmatrix}$$

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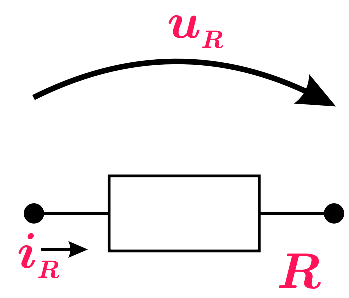
Fill-in Patterns

Resistor



Sparse tableau analysis

$$\begin{pmatrix} & \vdots & & \vdots & \\ \cdots & -1 & \cdots & \frac{1}{R} & \cdots \\ & \vdots & & \vdots & \end{pmatrix} \cdot \begin{pmatrix} \vdots \\ i_R \\ \vdots \\ u_R \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \end{pmatrix}$$

Superposition of matrices

$$p_i \cdot \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{pmatrix}$$

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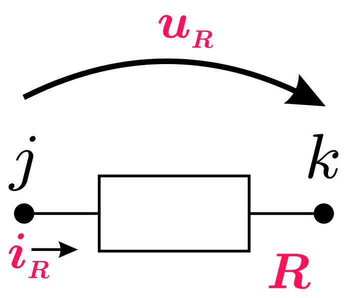
Fill-in Patterns

Resistor



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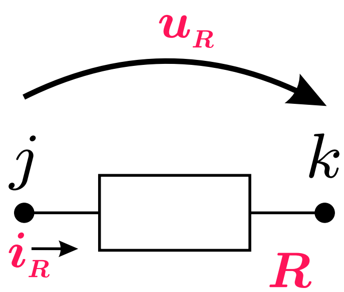
Fill-in Patterns

Resistor



Modified nodal analysis

$$\begin{pmatrix} & \vdots & & \vdots & & \vdots & & \vdots & \\ \dots & -1 & \dots & 0 & \dots & \frac{1}{R} & \dots & -\frac{1}{R} & \dots \\ & \vdots & & \vdots & & \vdots & & \vdots & \\ \dots & 0 & \dots & -1 & \dots & -\frac{1}{R} & \dots & \frac{1}{R} & \dots \\ & \vdots & & \vdots & & \vdots & & \vdots & \end{pmatrix} \cdot \begin{pmatrix} \vdots \\ i_j \\ \vdots \\ i_k \\ \vdots \\ u_j \\ \vdots \\ u_k \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \end{pmatrix}$$

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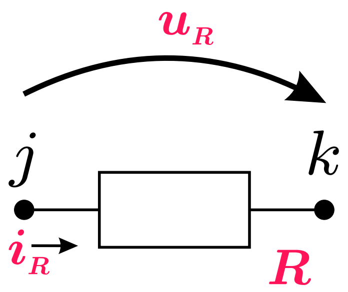
Fill-in Patterns

Resistor



Modified nodal analysis

$$\begin{pmatrix} & \vdots & & \vdots & & \vdots & & \vdots & \\ \cdots & -1 & \cdots & 0 & \cdots & \frac{1}{R} & \cdots & -\frac{1}{R} & \cdots \\ & \vdots & & \vdots & & \vdots & & \vdots & \\ \cdots & 0 & \cdots & -1 & \cdots & -\frac{1}{R} & \cdots & \frac{1}{R} & \cdots \\ & \vdots & & \vdots & & \vdots & & \vdots & \end{pmatrix} \cdot \begin{pmatrix} \vdots \\ i_j \\ \vdots \\ i_k \\ \vdots \\ u_j \\ \vdots \\ u_k \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \end{pmatrix}$$

General case

$$\left(\mathbf{A}_0 + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\top})\right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}_0 + \sum_j s_j \cdot \mathbf{r}_j$$

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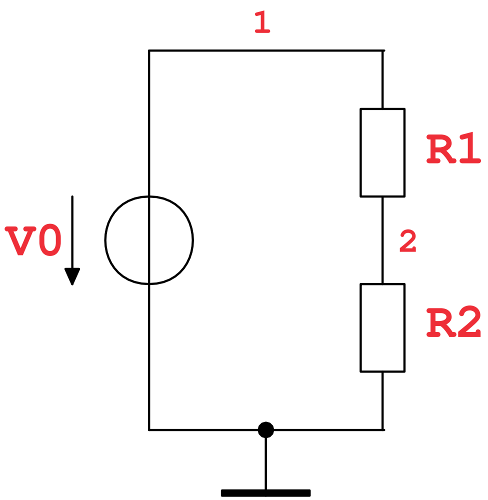
Resistive Networks



$$\left(\mathbf{A}_0 + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\top})\right) \cdot \mathbf{x} = \mathbf{r} \qquad p_i \in [-\Delta p_i, \Delta p_i]$$

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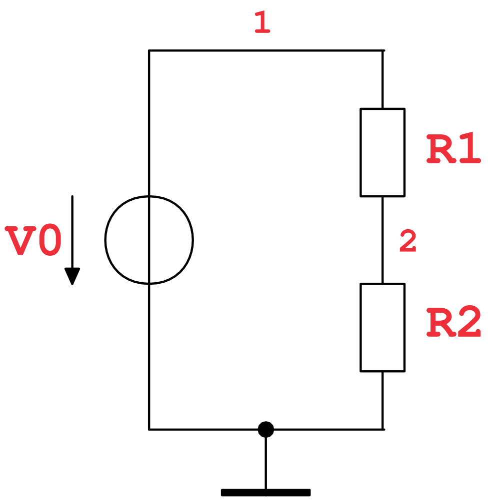
Resistive Networks



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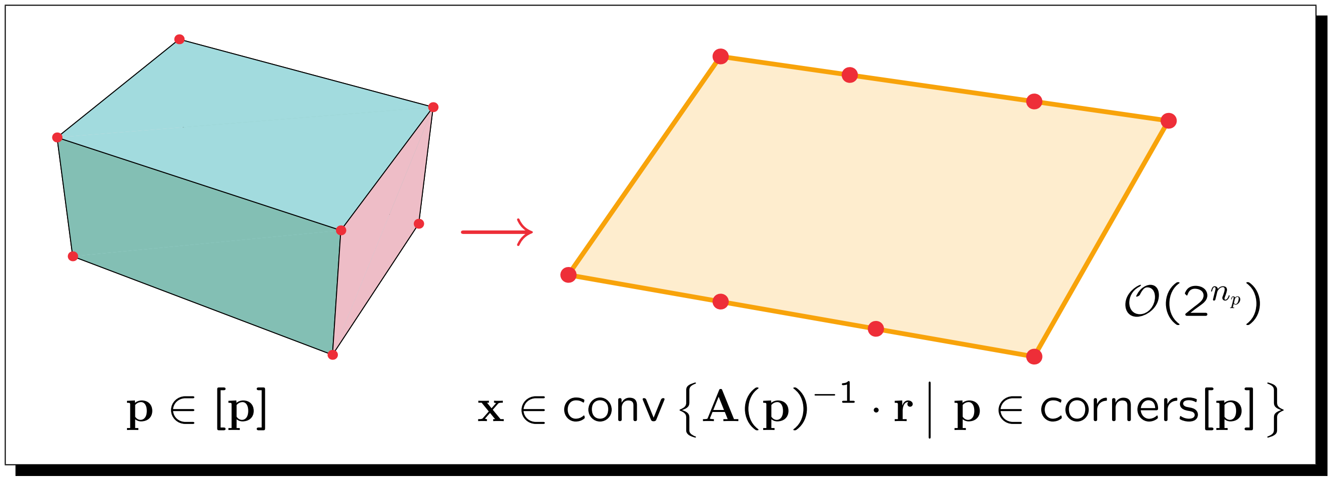

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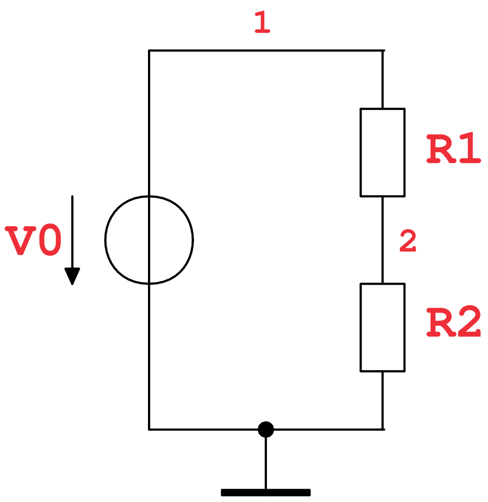
Resistive Networks



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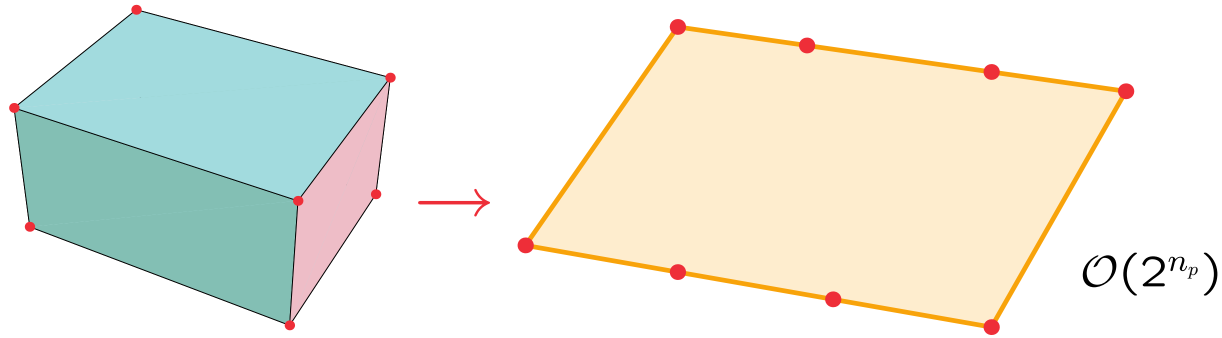

$\mathbf{p} \in [\mathbf{p}]$ $\qquad$ $\mathbf{x} \in \operatorname{conv}\left\{\mathbf{A}(\mathbf{p})^{-1} \cdot \mathbf{r} \,\middle|\, \mathbf{p} \in \operatorname{corners}[\mathbf{p}]\right\}$

Regularity condition $\qquad 0 \neq \operatorname{sign} \det \mathbf{A}(\mathbf{p}) = \text{const} \qquad$ for $\mathbf{p} \in \operatorname{corners}[\mathbf{p}]$

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Resistive Networks: Quick variant

$$\left(\mathbf{A}_0 + \sum_{i \neq j} p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\mathsf{T}}) + p_j \cdot (\mathbf{u}_j \cdot \mathbf{v}_j^{\mathsf{T}})\right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

$$p_i \in [-\Delta p_i, \Delta p_i]$$

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Resistive Networks: Quick variant

$$\underbrace{\left(\mathbf{A}_0 + \sum_{i \neq j} p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\top})}_{\mathbf{B}} + p_j \cdot (\mathbf{u}_j \cdot \mathbf{v}_j^{\top})\right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

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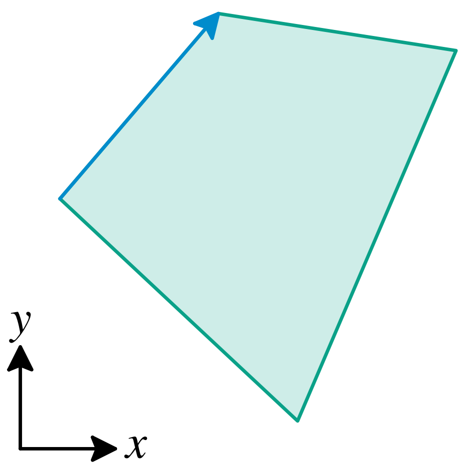
Resistive Networks: Quick variant



$$\underbrace{\left(\mathbf{A}_0 + \sum_{i \neq j} p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\mathsf{T}}) \right.}_{\mathbf{B}} \left. + p_j \cdot (\mathbf{u}_j \cdot \mathbf{v}_j^{\mathsf{T}}) \right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r} \qquad p_i \in [-\triangle p_i, \triangle p_i]$$

Sherman-Morrison formula

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^{\mathsf{T}} \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^{\mathsf{T}} \mathbf{B}^{-1} \right) \mathbf{r}$$

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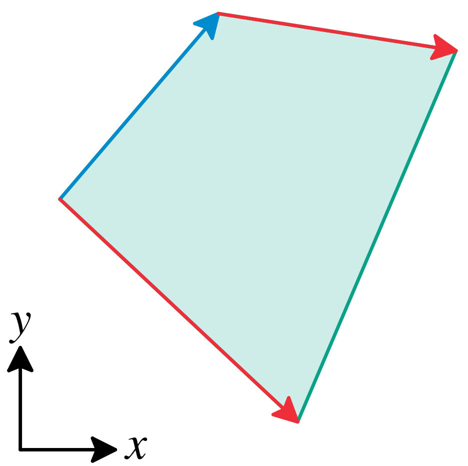
Resistive Networks: Quick variant



$$\underbrace{\left(\mathbf{A}_0 + \sum_{i \neq j} p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\top)\right.}_{\mathbf{B}} \left. + p_j \cdot (\mathbf{u}_j \cdot \mathbf{v}_j^\top)\right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

$$p_i \in [-\triangle p_i, \triangle p_i]$$

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$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^\top \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^\top \mathbf{B}^{-1} \right) \mathbf{r}$$

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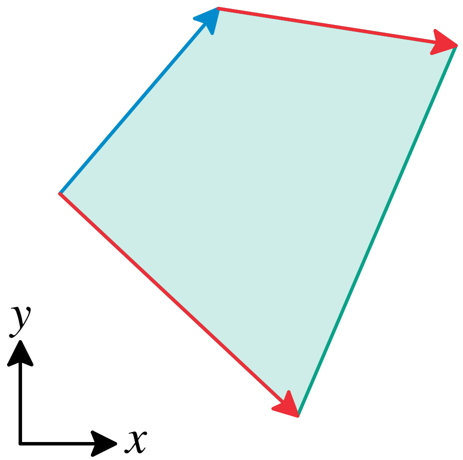
Resistive Networks: Quick variant



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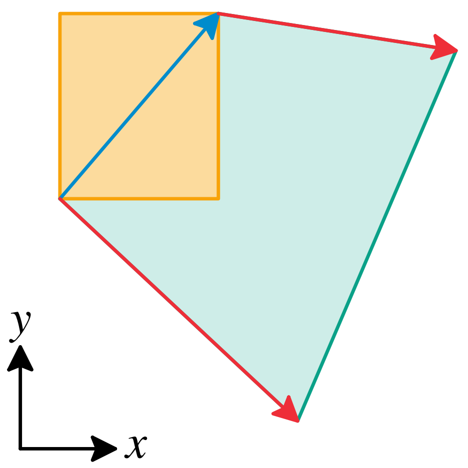
Resistive Networks: Quick variant

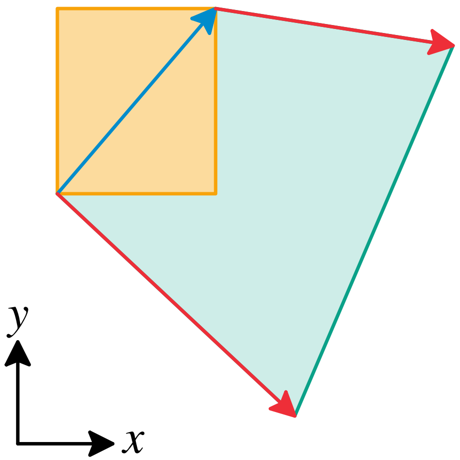


$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^{\intercal} \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^{\intercal} \mathbf{B}^{-1} \right) \mathbf{r}$$

$$[\tilde{\mathbf{r}}] := \mathbf{A}_0^{-1} \cdot \mathbf{r}$$
$$[\tilde{\mathbf{u}}_j] := \mathbf{A}_0^{-1} \cdot \mathbf{u}_j$$

$$[y] = 1/[p_j] + \mathbf{v}_j^{\intercal} \cdot [\tilde{\mathbf{u}}_j]$$

Resistive Networks: Quick variant



$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^{\mathsf{T}} \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^{\mathsf{T}} \mathbf{B}^{-1} \right) \mathbf{r}$$

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$$[y] = 1/[p_j] + \mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{u}}_j]$$

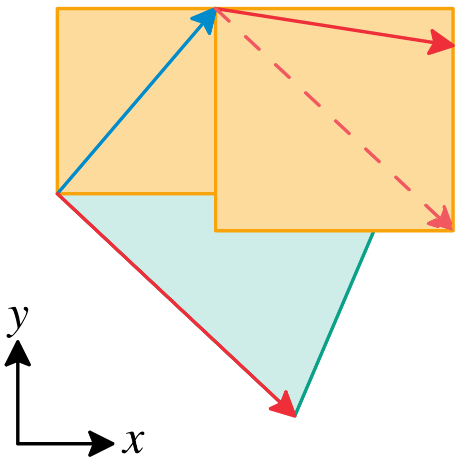
Regularity test

If $[y] \not\ni 0$

Interval-valued Sherman-Morrison

$$[\tilde{\mathbf{r}}] \leftarrow [\tilde{\mathbf{r}}] - \frac{1}{[y]} \cdot \left([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{r}}]) \right)$$

Resistive Networks: Quick variant



Regularity test

Interval-valued Sherman-Morrison

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^{\mathsf{T}} \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^{\mathsf{T}} \mathbf{B}^{-1} \right) \mathbf{r}$$

Initializing

$$[\tilde{\mathbf{r}}] := \mathbf{A}_0^{-1} \cdot \mathbf{r}$$
$$[\tilde{\mathbf{u}}_j] := \mathbf{A}_0^{-1} \cdot \mathbf{u}_j, \quad j = 1, \dots, n_p$$

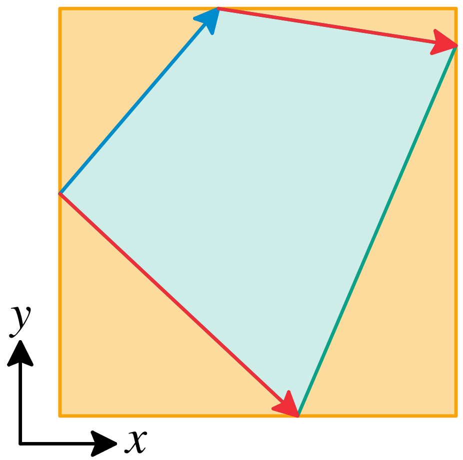
For $j = 1, \dots, n_p$

$$[y] = 1/[p_j] + \mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{u}}_j]$$

If $[y] \not\ni 0$

$$[\tilde{\mathbf{r}}] \leftarrow [\tilde{\mathbf{r}}] - \tfrac{1}{[y]} \cdot \left([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{r}}]) \right)$$
$$[\tilde{\mathbf{u}}_i] \leftarrow [\tilde{\mathbf{u}}_i] - \tfrac{1}{[y]} \cdot \left([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{u}}_i]) \right), \; i > j$$

Resistive Networks: Quick variant



$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{B}^{-1} \cdot \mathbf{r} - \left(\frac{1}{1/p_j + \mathbf{v}_j^{\mathsf{T}} \mathbf{B}^{-1} \mathbf{u}_j} \mathbf{B}^{-1} \mathbf{u}_j \mathbf{v}_j^{\mathsf{T}} \mathbf{B}^{-1} \right) \mathbf{r}$$

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$[\tilde{\mathbf{u}}_j] := \mathbf{A}_0^{-1} \cdot \mathbf{u}_j, \quad j = 1, \dots, n_p$

For $j = 1, \dots, n_p$

$[y] = 1/[p_j] + \mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{u}}_j]$

Regularity test

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$[\tilde{\mathbf{r}}] \leftarrow [\tilde{\mathbf{r}}] - \frac{1}{[y]} \cdot \left([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{r}}]) \right)$

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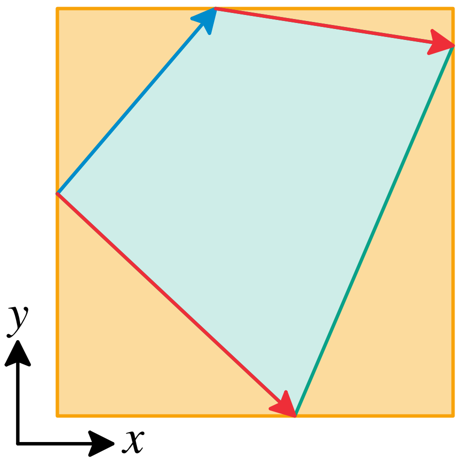
Resistive Networks: Quick variant



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$[\tilde{\mathbf{r}}] := \mathbf{A}_0^{-1} \cdot \mathbf{r}$

$[\tilde{\mathbf{u}}_j] := \mathbf{A}_0^{-1} \cdot \mathbf{u}_j, \quad j = 1, \ldots, n_p$

For $j = 1, \ldots, n_p$

$[y] = 1/[p_j] + \mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{u}}_j]$

If $[y] \not\ni 0$

$[\tilde{\mathbf{r}}] \leftarrow [\tilde{\mathbf{r}}] - \frac{1}{[y]} \cdot \left([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{r}}]) \right)$

$[\tilde{\mathbf{u}}_i] \leftarrow [\tilde{\mathbf{u}}_i] - \frac{1}{[y]} \cdot \left([\tilde{\mathbf{u}}_j] \cdot (\mathbf{v}_j^{\mathsf{T}} \cdot [\tilde{\mathbf{u}}_i]) \right), \ i > j$

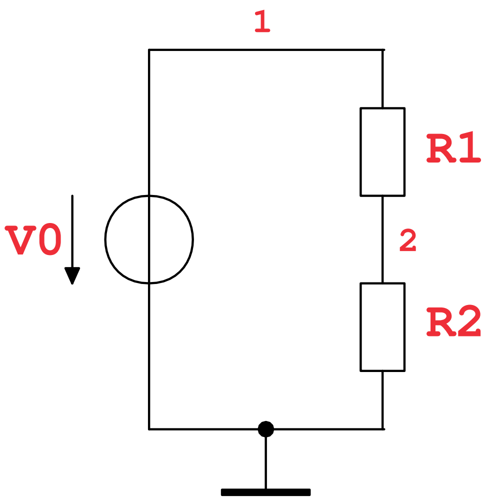
$\mathcal{O}(n_p^2)$ "payed" by loss of accuracy

- **Wrapping effect**
  $[\mathbf{x}] \supseteq \text{conv}\{\mathbf{x}_\nu\}_\nu$
- **Dependencies**
  $[\tilde{\mathbf{u}}_\nu] \leftrightarrow [\tilde{\mathbf{u}}_\mu]$

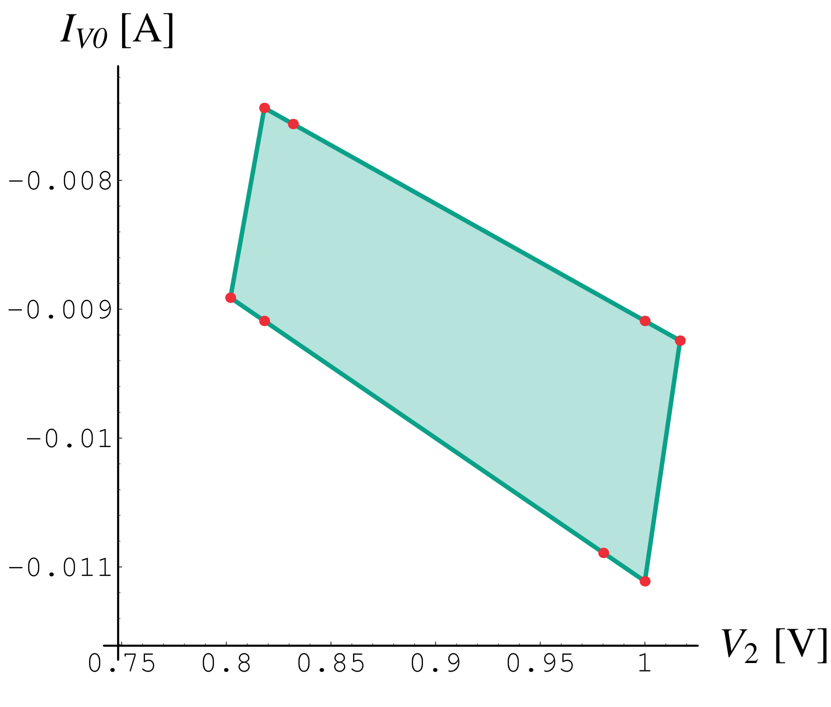
Regularity test

Interval-valued Sherman-Morrison

Resistive Networks



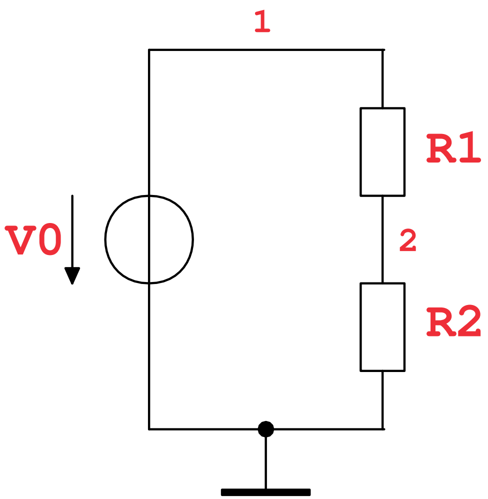
— conv(⋱ . ⋱ .)

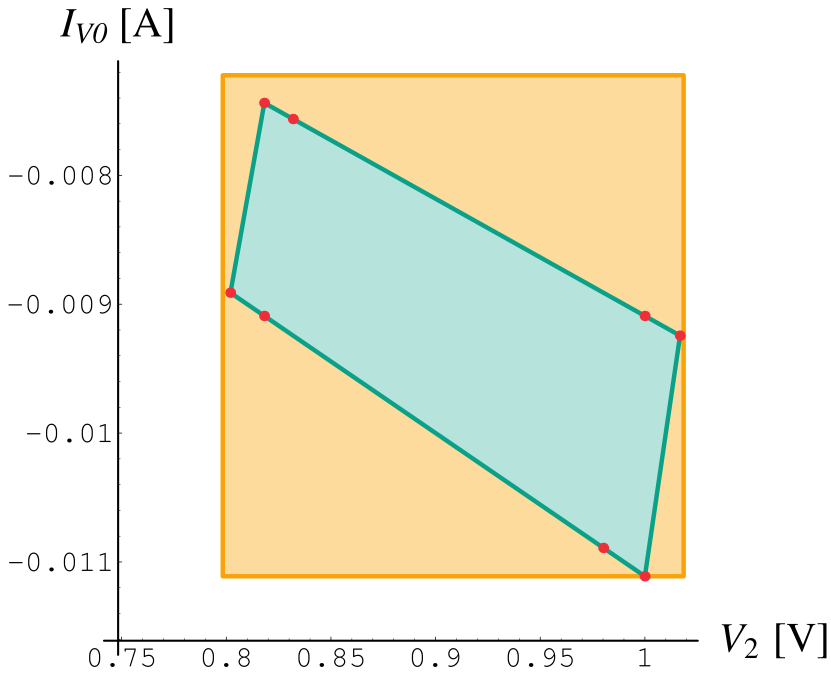


V0 $= 1\,\mathrm{V}$
R1 $= 10\,\Omega$
R2 $= 100\,\Omega$

Tolerance $\pm 10\%$

Resistive Networks



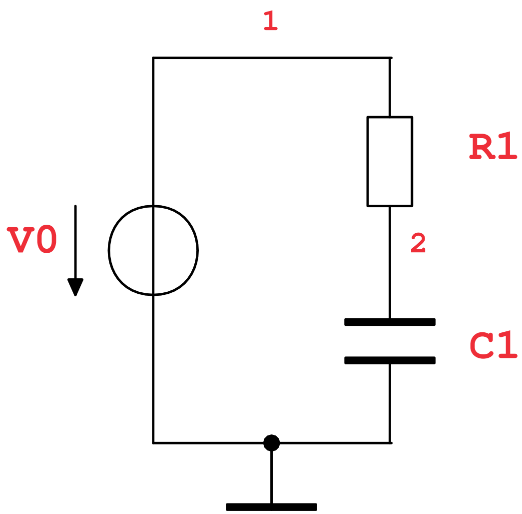


— conv(⋱, ⋱)
— Quick variant

V0 $= 1\,\mathrm{V}$
R1 $= 10\,\Omega$
R2 $= 100\,\Omega$

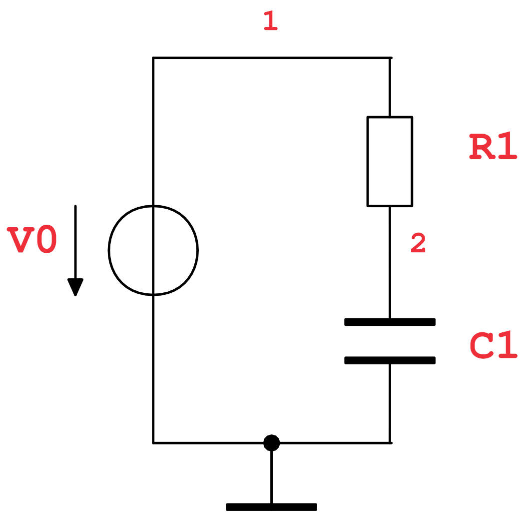
Tolerance $\pm 10\%$

RLC Networks



$$\begin{pmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \frac{1}{R_1} + C_1\, s & 0 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ I_{V_0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_0 \end{pmatrix} \qquad s = 2\pi i f \in \mathbb{C}$$

RLC Networks



$$\begin{pmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ -\frac{1}{R_1} & \frac{1}{R_1} + C_1\, s & 0 \\ 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ I_{V_0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_0 \end{pmatrix} \qquad s = 2\pi i f \in \mathbb{C}$$

real representation

$$\begin{pmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 1 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} & 0 & 0 & -C_1\, 2\pi f & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_1} & -\frac{1}{R_1} & 1 \\ 0 & C_1\, 2\pi f & 0 & -\frac{1}{R_1} & \frac{1}{R_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \mathrm{Re}\, V_1 \\ \mathrm{Re}\, V_2 \\ \mathrm{Re}\, I_{V_0} \\ \mathrm{Im}\, V_1 \\ \mathrm{Im}\, V_2 \\ \mathrm{Im}\, I_{V_0} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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RLC Networks

Fill-in patterns

$$\left(\mathbf{A} + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\mathsf{T}}) + \sum_i p_i \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^{\mathsf{T}})\right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

Two patterns per parameter

RLC Networks

Fill-in patterns

$$\left(\mathbf{A} + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\intercal) + \sum_i p_i \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^\intercal)\right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

Two patterns per parameter

Outer approximation

$$\left(\mathbf{A} + \sum_i p_i \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^\intercal) + \sum_i \breve{p}_i \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^\intercal)\right) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{r}$$

Independent treatment

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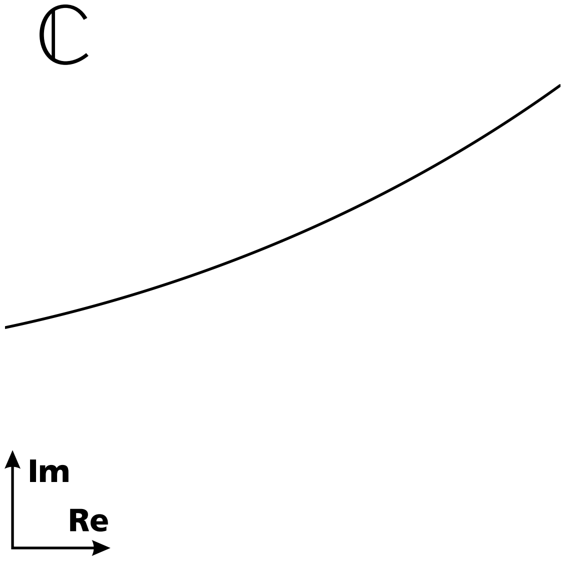

$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\mathsf{T}}) + p \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^{\mathsf{T}})) \cdot \mathbf{x} = \mathbf{r}$$

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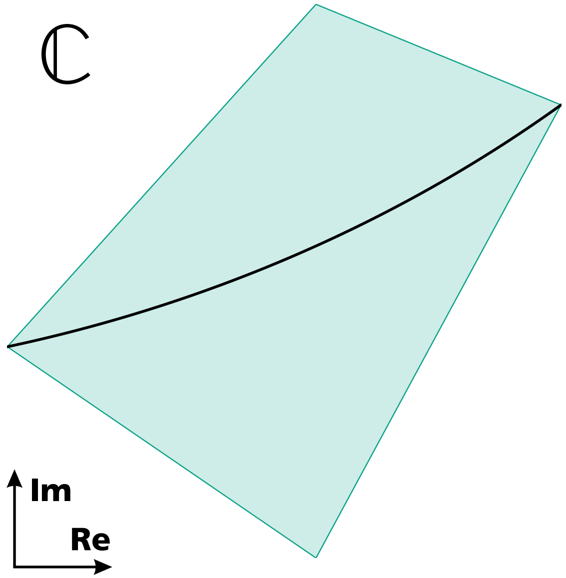
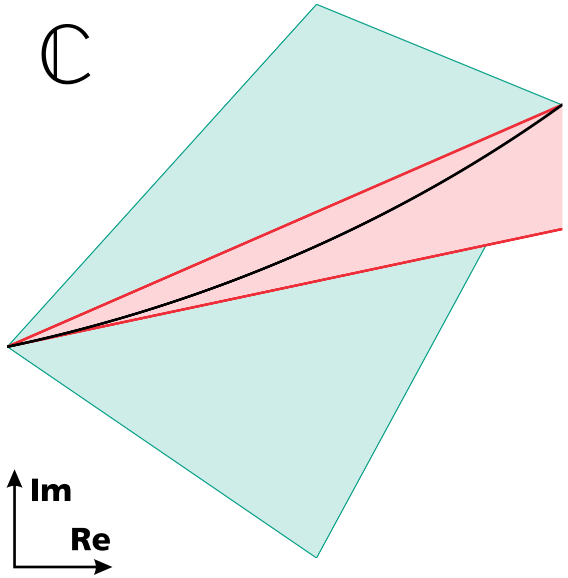


$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\top}) + p \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^{\top})) \cdot \mathbf{x} = \mathbf{r}$$

$$\mathbf{A}(p, q) := \mathbf{A}_0 + p \cdot (\mathbf{u} \cdot \mathbf{v}^{\top}) + q \cdot (\check{\mathbf{u}} \cdot \check{\mathbf{v}}^{\top}),\ \text{regular}\ p, q \in [-\triangle p, \triangle p]$$

$$\mathbf{x} \in \operatorname{conv} \left\{ \mathbf{A}(-\triangle p, -\triangle p)^{-1}\mathbf{r},\ \mathbf{A}(-\triangle p, \triangle p)^{-1}\mathbf{r},\ \mathbf{A}(\triangle p, \triangle p)^{-1}\mathbf{r},\ \mathbf{A}(\triangle p, -\triangle p)^{-1}\mathbf{r} \right\}$$

RLC Networks



$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\top}) + p \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^{\top})) \cdot \mathbf{x} = \mathbf{r}$$

$\mathbf{A}(p,q) := \mathbf{A}_0 + p \cdot (\mathbf{u} \cdot \mathbf{v}^{\top}) + q \cdot (\check{\mathbf{u}} \cdot \check{\mathbf{v}}^{\top})$, regular $p, q \in [-\Delta p, \Delta p]$

$$\mathbf{x} \in \mathrm{conv}\left\{ \mathbf{A}(-\Delta p, -\Delta p)^{-1}\mathbf{r},\ \mathbf{A}(-\Delta p, \Delta p)^{-1}\mathbf{r},\ \mathbf{A}(\Delta p, \Delta p)^{-1}\mathbf{r},\ \mathbf{A}(\Delta p, -\Delta p)^{-1}\mathbf{r} \right\}$$

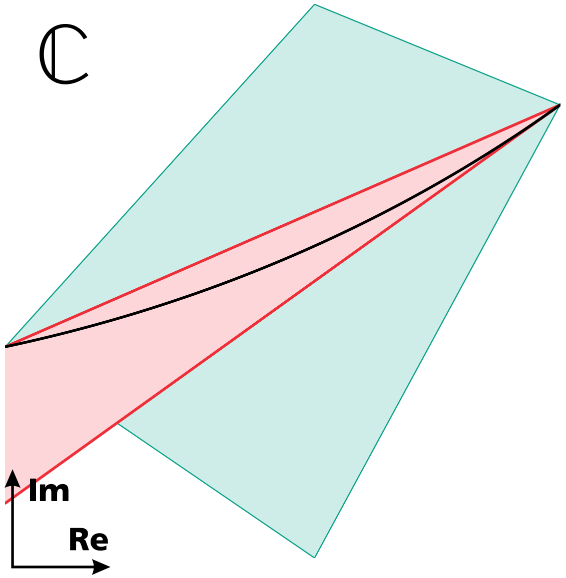
$$\mathbf{x} \in \left\{ \mathbf{A}(-\Delta p, -\Delta p)^{-1}\mathbf{r} + \lambda \cdot \mathbf{d}_1 + \mu \cdot \mathbf{t}_1 \,\middle|\, \lambda, \mu \geq 0 \right\}$$

$\mathbf{d}_1$ diagonal

$\mathbf{t}_1$ tangent

Generalized Sherman-Morrison formula

RLC Networks



$$(\mathbf{A}_0 + p\cdot(\mathbf{u}_i\cdot\mathbf{v}_i^{\top}) + p\cdot(\check{\mathbf{u}}_i\cdot\check{\mathbf{v}}_i^{\top}))\cdot\mathbf{x} = \mathbf{r}$$

$\mathbf{A}(p,q) := \mathbf{A}_0 + p\cdot(\mathbf{u}\cdot\mathbf{v}^{\top}) + q\cdot(\check{\mathbf{u}}\cdot\check{\mathbf{v}}^{\top})$, regular $p, q \in [-\Delta p, \Delta p]$

$$\mathbf{x} \in \text{conv}\left\{ \mathbf{A}(-\Delta p, -\Delta p)^{-1}\mathbf{r} \quad \mathbf{A}(-\Delta p, \Delta p)^{-1}\mathbf{r} \quad \mathbf{A}(\Delta p, \Delta p)^{-1}\mathbf{r} \quad \mathbf{A}(\Delta p, -\Delta p)^{-1}\mathbf{r} \right\}$$

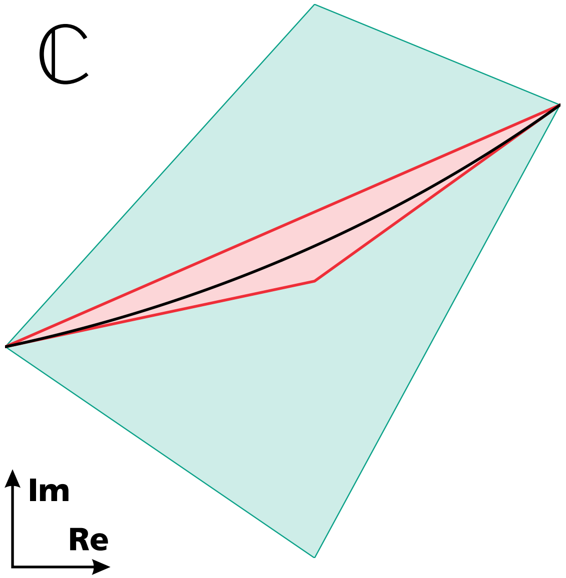
Generalized
Sherman-Morrison formula

$$\mathbf{x} \in \left\{ \mathbf{A}(\Delta p, \Delta p)^{-1}\mathbf{r} + \lambda\cdot\mathbf{d}_2 + \mu\cdot\mathbf{t}_2 \,\middle|\, \lambda, \mu \geq 0 \right\}$$

$\mathbf{d}_2$ diagonal

$\mathbf{t}_2$ tangent

RLC Networks



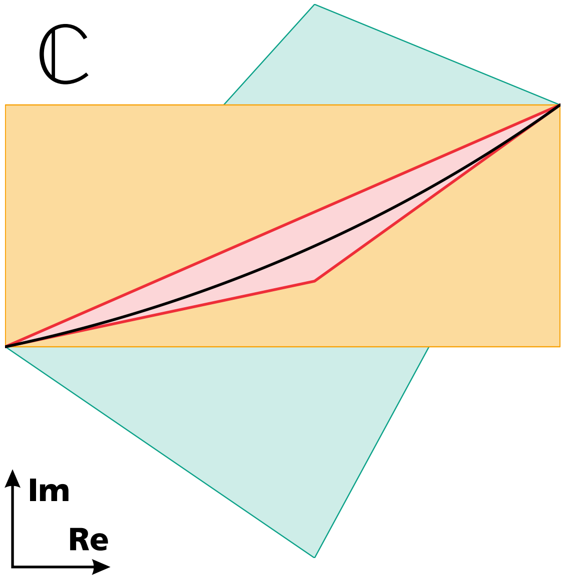
$$(\mathbf{A}_0 + p \cdot (\mathbf{u}_i \cdot \mathbf{v}_i^{\mathsf{T}}) + p \cdot (\check{\mathbf{u}}_i \cdot \check{\mathbf{v}}_i^{\mathsf{T}})) \cdot \mathbf{x} = \mathbf{r}$$

$\mathbf{A}(p, q) := \mathbf{A}_0 + p \cdot (\mathbf{u} \cdot \mathbf{v}^{\mathsf{T}}) + q \cdot (\check{\mathbf{u}} \cdot \check{\mathbf{v}}^{\mathsf{T}})$, regular $p, q \in [-\Delta p, \Delta p]$

$$\mathbf{x} \in \operatorname{conv} \left\{ \mathbf{A}(-\Delta p, \Delta p)^{-1}\mathbf{r},\ \mathbf{A}(\Delta p, \Delta p)^{-1}\mathbf{r},\ \mathbf{A}(-\Delta p, -\Delta p)^{-1}\mathbf{r},\ \mathbf{A}(\Delta p, -\Delta p)^{-1}\mathbf{r} \right\}$$

$$\mathbf{x} \in \operatorname{conv} \left\{ \mathbf{A}(-\Delta p, -\Delta p)^{-1}\mathbf{r},\ \mathbf{A}(\Delta p, \Delta p)^{-1}\mathbf{r},\ \tfrac{1}{2}\left(\mathbf{A}(-\Delta p, \Delta p)^{-1}\mathbf{r} + \mathbf{A}(\Delta p, -\Delta p)^{-1}\mathbf{r}\right) \right\}$$

RLC Networks



$$(\mathbf{A}_0 + p\cdot(\mathbf{u}_i\cdot\mathbf{v}_i^{\top}) + p\cdot(\check{\mathbf{u}}_i\cdot\check{\mathbf{v}}_i^{\top}))\cdot\mathbf{x} = \mathbf{r}$$

$$\mathbf{A}(p,q) := \mathbf{A}_0 + p\cdot(\mathbf{u}\cdot\mathbf{v}^{\top}) + q\cdot(\check{\mathbf{u}}\cdot\check{\mathbf{v}}^{\top}),\ \text{regular } p,q\in[-\triangle p,\triangle p]$$

$$\mathbf{x}\in \operatorname{conv}\left\{ \mathbf{A}(-\triangle p,-\triangle p)^{-1}\mathbf{r},\ \mathbf{A}(-\triangle p,\triangle p)^{-1}\mathbf{r},\ \mathbf{A}(\triangle p,\triangle p)^{-1}\mathbf{r},\ \mathbf{A}(\triangle p,-\triangle p)^{-1}\mathbf{r} \right\}$$

$$[\mathbf{x}] \supseteq \operatorname{conv}\left\{ \mathbf{A}(-\triangle p,-\triangle p)^{-1}\mathbf{r},\ \mathbf{A}(\triangle p,\triangle p)^{-1}\mathbf{r},\ \tfrac{1}{2}\left(\mathbf{A}(-\triangle p,\triangle p)^{-1}\mathbf{r} + \mathbf{A}(\triangle p,-\triangle p)^{-1}\mathbf{r}\right) \right\}$$

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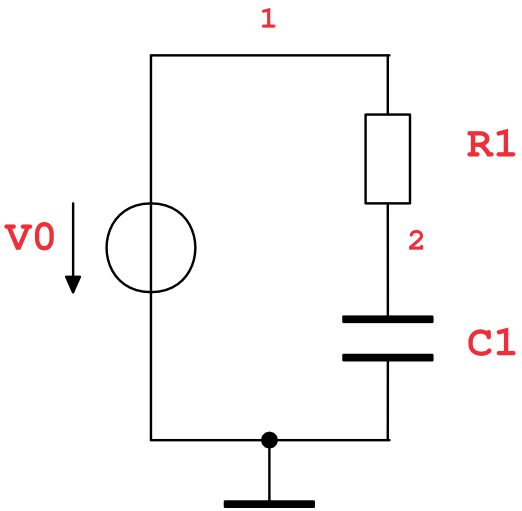
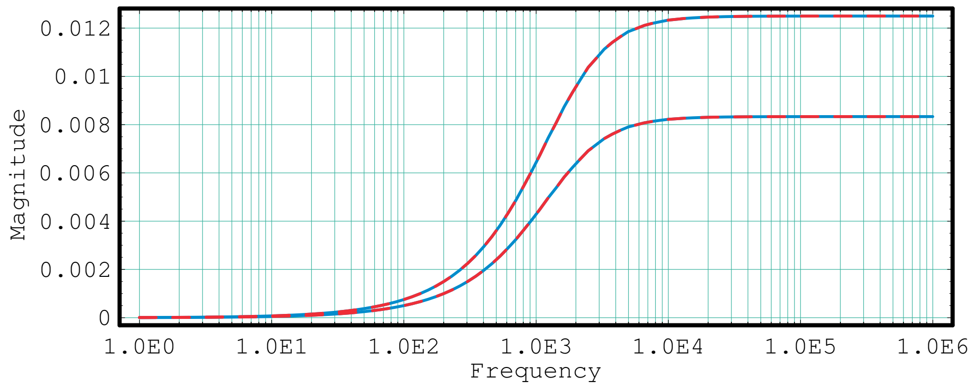
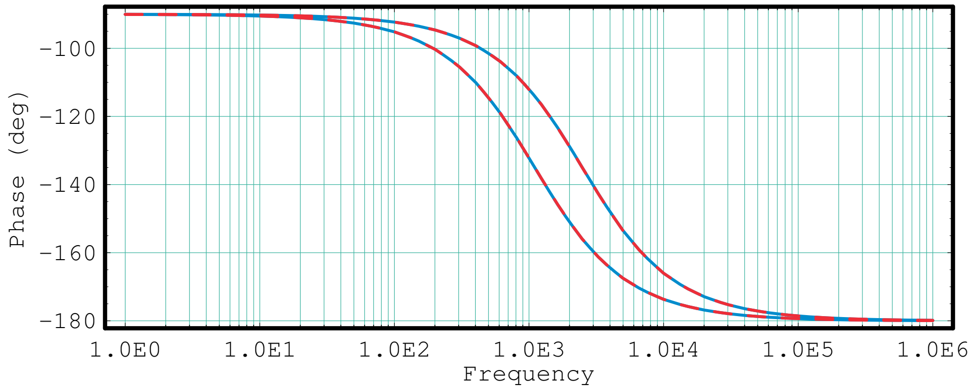
RLC Networks







— Inner approx. for I_{V_0}

— **IntervalACAnalysis**

V1 $= 1\,\mathrm{V}$

R1 $= 100\,\Omega \pm 20\%$

C1 $= 1\,\mu\mathrm{F} \pm 20\%$

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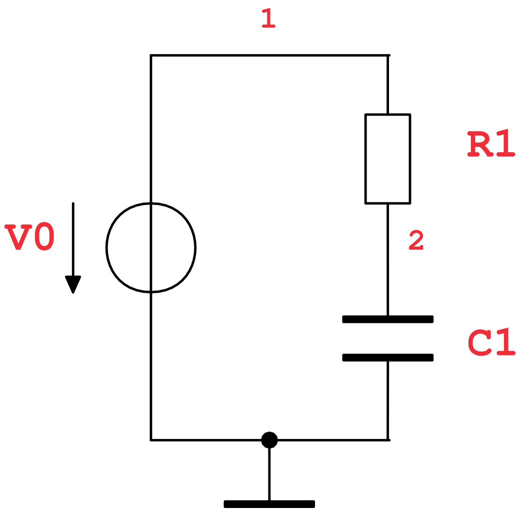

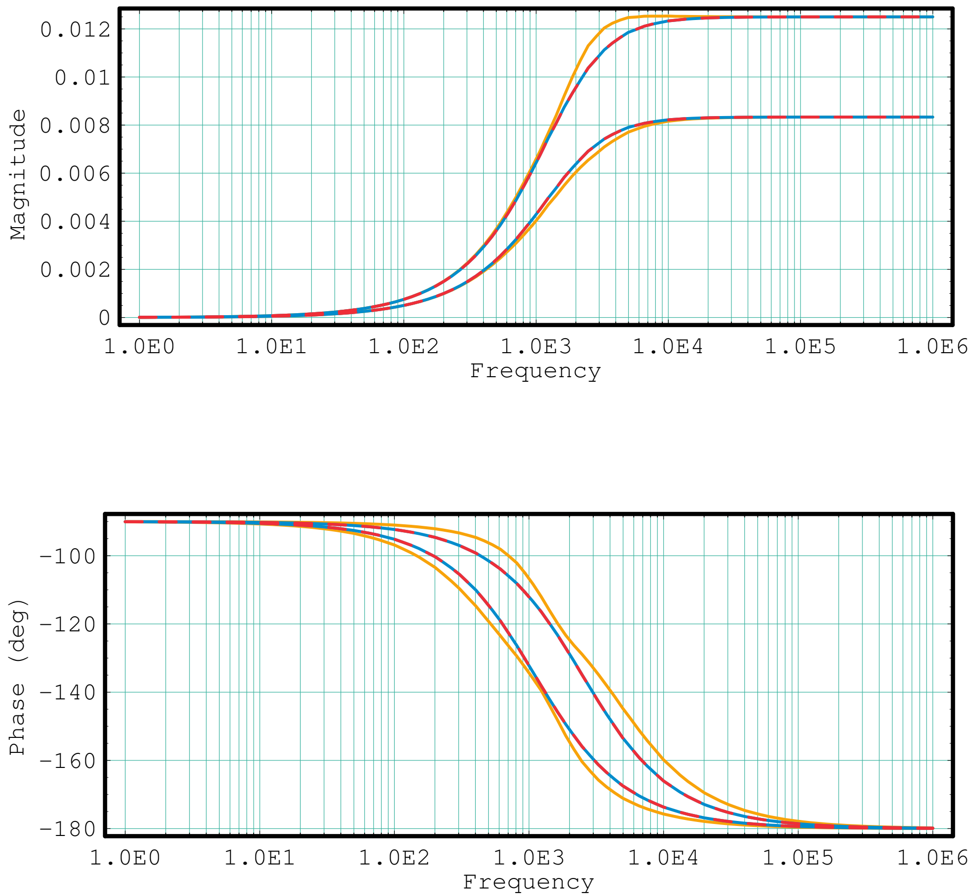

V1 $= 1\,\mathrm{V}$
R1 $= 100\,\Omega \pm 20\%$
C1 $= 1\,\mu\mathrm{F} \pm 20\%$

— Inner approx. for I_{V_0}
— **IntervalACAnalysis**
— **QuickACAnalysis**

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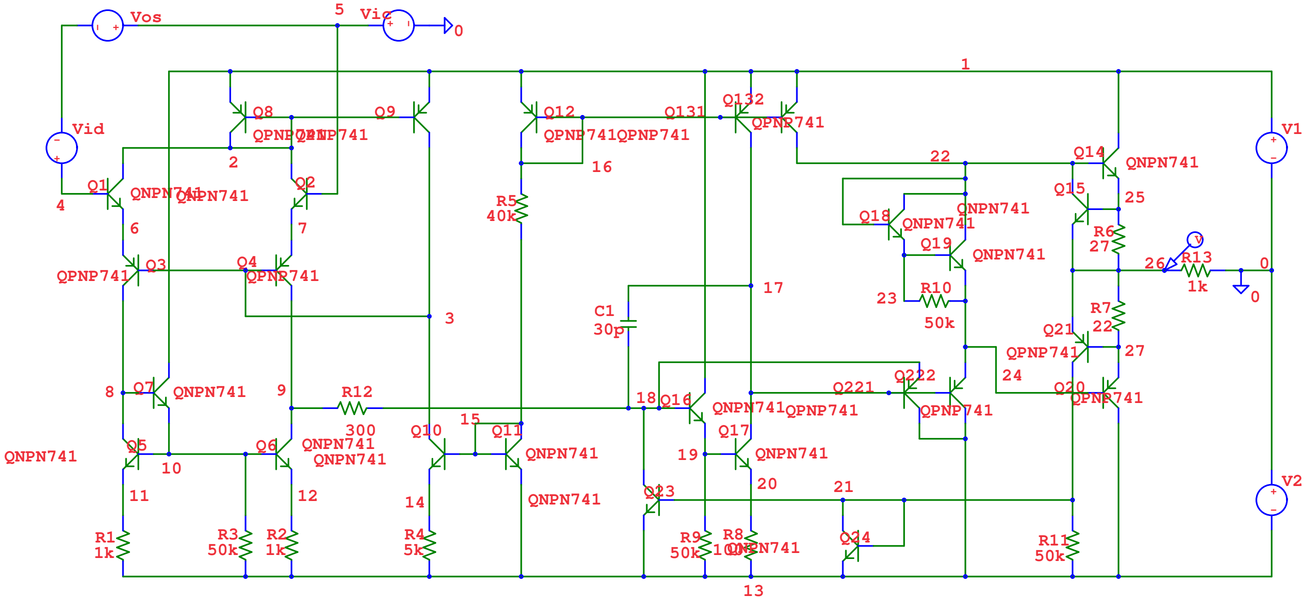
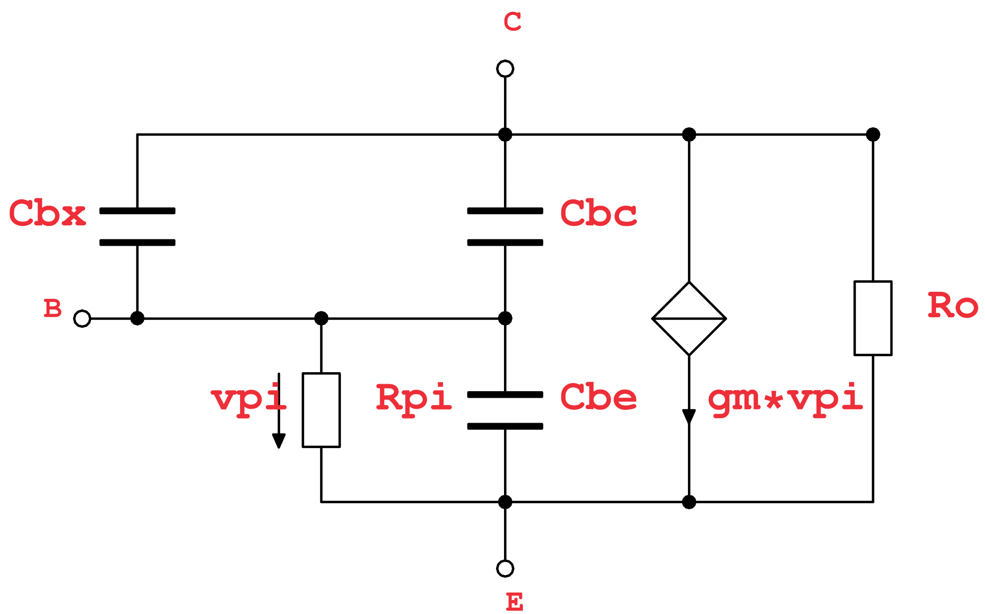
AC-Analysis of an Operational Amplifier

Bound the output voltage for the frequency range from 0.1 Hz to 1 MHz (constant operating point)



AC-Analysis of an Operational Amplifier

Simple linear model
for bipolar transistor



Tolerance specification
(capacitances) $\pm 20\%$

Number of transistors: 26
In sum 79 capacitors (internal and external)

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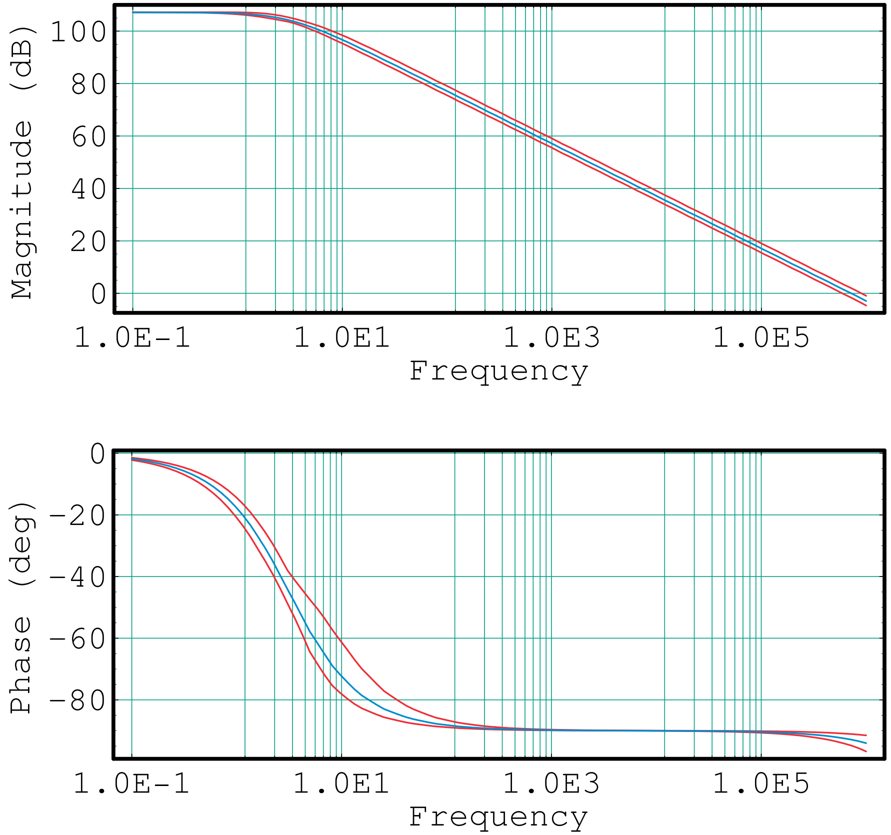
AC-Analysis of an Operational Amplifier

Bound the output voltage for the frequency range from 0.1 Hz to 1 MHz (constant operating point)

Tolerance specification (capacitances) ±20%



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Summary

Linear circuit analysis

- Method for treatment of large component tolerances and industrial-sized circuits
- Extension to complex-valued systems (frequency domain)
- Support of several circuit formulations (sparse tableau analysis, modified nodal analysis)

Implementation

- Combined symbolic/numerical approach (Mathematica, C++)
- Add-on to ITWM's commercial circuit analysis tool Analog Insydes